



National
Qualifications
2023

2023 Applications of Mathematics

Higher

Finalised Marking Instructions

© Scottish Qualifications Authority 2023

These marking instructions have been prepared by examination teams for use by SQA appointed markers when marking external course assessments.

The information in this document may be reproduced in support of SQA qualifications only on a non-commercial basis. If it is reproduced, SQA must be clearly acknowledged as the source. If it is to be reproduced for any other purpose, written permission must be obtained from permissions@sqa.org.uk.



General marking principles for Higher Applications of Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

generic scheme – this indicates why each mark is awarded

illustrative scheme – this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

- (a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.
- (b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.
- (c) One mark is available for each •. There are no half marks.
- (d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.
- (e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.
- (f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.
- (g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

- (h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 1 \text{ or } 3$$

(i) **Horizontal/vertical marking**

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

$$\begin{array}{cc} \bullet^5 & \bullet^6 \\ \bullet^5 & x = 2 \quad x = -4 \\ \bullet^6 & y = 5 \quad y = -7 \end{array}$$

Horizontal: $\bullet^5 x = 2 \text{ and } x = -4$
 $\bullet^6 y = 5 \text{ and } y = -7$

Vertical: $\bullet^5 x = 2 \text{ and } y = 5$
 $\bullet^6 x = -4 \text{ and } y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

- (j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

- (k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.
- (l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:
- working subsequent to a correct answer
 - correct working in the wrong part of a question
 - legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
 - omission of units
 - bad form (bad form only becomes bad form if subsequent working is correct), for example

$(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

$(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

$= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

gains full credit

- repeated error within a question, but not between questions or papers
- (m) In any ‘Show that...’ question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.
- (n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate’s response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.
- (o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.
- (p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

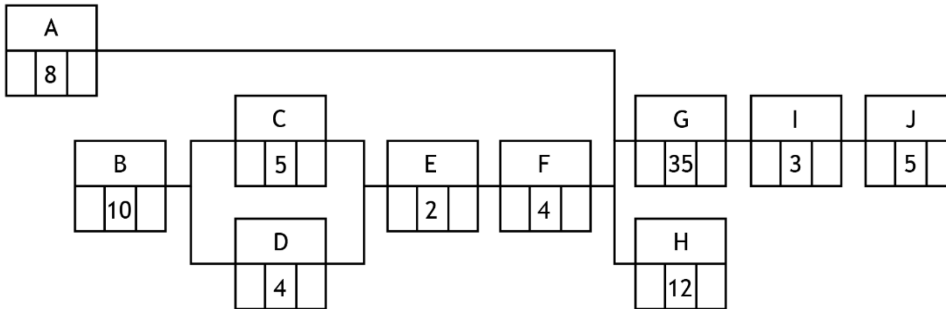
Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ state an assumption about number of years of schooling •² state an assumption on number of hours per day •³ estimate total number of hours	•¹ 11 – 15 (years) •² 5 – 8 (hours) $5 \times 190 \times 11 = 10450$ (hours) •³ eg OR $8 \times 190 \times 15 = 22800$ (hours)	3
Notes: 1. If 190 days is not used explicitly then some justification for the number of days used must be evident for •³ 2. Where an annual total has been calculated only •² is available. 3. For a valid calculation with no assumptions stated, award •³ only.					
Commonly Observed Responses: Candidate A 32 hours per week. 190 days = 38 weeks. $32 \times 38 \times$ number of years. Award 3/3					

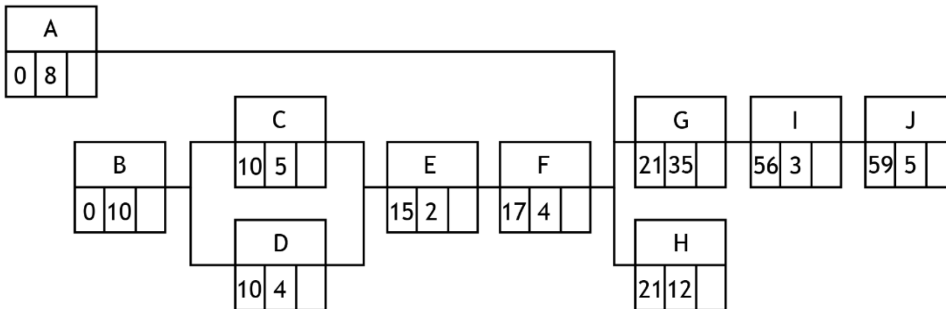
Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•¹ first six tasks and durations in correct sequence •² remaining four tasks and durations in correct sequence •³ forward scan from A-F correct •⁴ forward scan for G-J consistent with previous working •⁵ backward scan J, I, H, G correct •⁶ backward scan for F, E, D, C, B, A consistent with previous working	•¹ see note 1 •² see note 1 •³ see note 2 •⁴ see note 2 •⁵ see note 3 •⁶ see note 3	6

Notes:

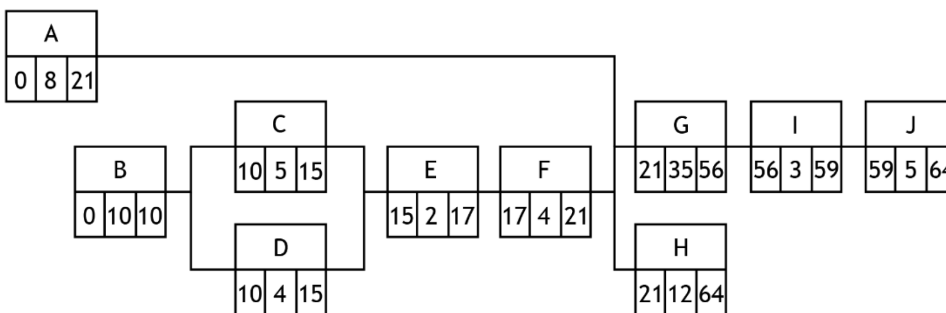
1. Letters and durations in correct order.



2. Forward scan.



3. Backward scan.



4. Ensure that letter H is checked carefully for the backward scan.

5. 80 is used as the latest completion time in task J, •⁵ is not available but •⁶ is still available as follow through.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(b)		•⁷ evidence of time constraint •⁸ conclusion with justification	•⁷ 8 minutes to complete task A or 16 minutes spare or 72 minutes earliest completion time •⁸ eg yes, because $16 > 8$ or 8 minutes left over	2

Notes:

1. Answer must be consistent with part (a).

Commonly Observed Responses:

Candidate A

Yes, because task A will take 8 minutes and there is 13 minutes float time. Award •⁷ only.

Candidate B

Yes, because there is 16 minutes left over. Award •⁷ only.

Candidate C

16 minutes to spare. 8 minutes delay. Yes, there is time to complete. Award •⁷ and •⁸.

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)	(i)	• ¹ state type of data	• ¹ discrete numerical or discrete quantitative	1
		(ii)	• ² state type of data	• ² (nominal) categorical or qualitative	1
Notes: 1. Do not accept any descriptions other than those above for • ² .					
Commonly Observed Responses:					
	(b)		• ³ give one reason • ⁴ give second reason	• ³ eg local survey and not a national survey • ⁴ eg not all voters will be able to access the survey online	2
Notes: 1. The above are illustrative solutions - candidates should be awarded marks for sensible answers. 2. Additional reasons - not targeting representative proportions of voting population; relying on people telling the truth					
Commonly Observed Responses: 1. For the award of • ³ and • ⁴ (a) Accept eg • people could vote multiple times online • invite goes into junk mail • not targeting representative proportions of voting population (b) Do not accept eg • people might not want to put their true political views online • people might change their mind • people lying/dishonest • small sample size					

Question			Generic scheme	Illustrative scheme	Max mark
4.			•¹ calculate taxable income after pension deductions •² calculate tax for starter, basic or intermediate band •³ calculate tax for remaining bands and total annual tax to pay •⁴ calculate annual net salary	•¹ £34,975.10 •² see table •³ see table (£4552.27) •⁴ £27,351.38	4

Notes:

1. Table of taxable income and tax

Mark 2

Mark 3

Taxable Income (£)	Tax Band	Tax Payable (£)
12,570	0%	0
2162	19%	410.78
10,956	20%	2191.20
9287.10	21%	1950.29
	Total	4552.27

2. For •² taxable incomes must be stated.

3. •² can be awarded when taxable incomes are not stated but working is shown, eg $(25,688 - 14,732) \times 0.2$

4. •² can be awarded for correctly working out tax in each band and total.

5. •⁴ can only be awarded if tax, National Insurance and pension contribution are all deducted.

6. Final answer must be stated to 2 decimal places, ignore any rounding errors or truncation.

Commonly Observed Responses:

Candidate A - candidate did not deduct pension contribution before calculating tax.

Mark 2

Mark 3

Taxable Income (£)	Tax Band	Tax Payable (£)
12,570	0%	0
2162	19%	410.78
10,956	20%	2191.20
11,012	21%	2312.52
	Total	4914.50

Leading to an annual net salary of £26,989.15. Award •², •³ and •⁴

Candidate B - deducted pension and National Insurance contributions before calculating tax.

Mark 2

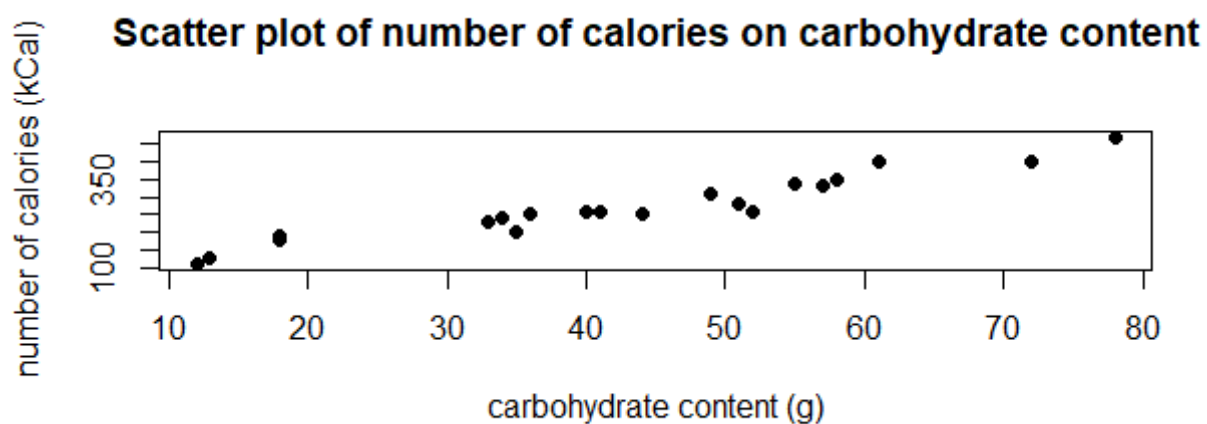
Mark 3

Taxable Income (£)	Tax Band	Tax Payable (£)
12,570	0%	0
2162	19%	410.78
10,956	20%	2191.20
6215.65	21%	1305.29
	Total	3907.27

Leading to an annual net salary of £27,996.38. Award •², •³ and •⁴.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•¹ generate scatter plot •² appropriate title and axes labels	•¹ (see below) •² (see below)	2

Notes:



Notes:

Commonly Observed Responses:

1. Candidates who plot carbohydrate content on calories and have the title calories on carbohydrate content - do not award •¹, award •².
2. Title must include both variables of the plot to award •².

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)	(i)	•³ generate correlation coefficient to determine correlation coefficient •⁴ state correlation coefficient for number of calories and carbohydrate content	•³ output from software (see note 1) •⁴ 0.962...	2
		(ii)	•⁵ appropriate comment	•⁵ eg this suggests there is a (strong) positive linear association between carbohydrate and calories	1

Notes:

1. R Output

Pearson's product-moment correlation

```
data: Carbohydrate.content and Number.of.calories
t = 15.941, df = 20, p-value = 7.8e-13
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.9110422 0.9847070
sample estimates:
   cor 0.9628262
```

- For •⁴ the correlation coefficient must be stated explicitly, separate from the output.
- Accept answers rounded or truncated to 2 decimal places for •⁴.
- If a candidate generates an R^2 value, they must state r to award •³, •⁴ and •⁵.
- For •⁵, accept 'positive linear' or 'positive correlation'.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(c)	(i)	•⁶ generate coefficient and intercept •⁷ state equation	•⁶ output from software (see note 1) •⁷ calories = $73.65 + 4.65 \times \text{carbohydrate}$	2
		(ii)	•⁸ appropriate comment for slope parameter •⁹ appropriate comment for intercept parameter	•⁸ eg the number of calories increases by 4.65 (kCal) for every additional 1 gram of carbohydrate •⁹ eg the number of calories in a drink with 0g carbohydrate content would be 73.65 (kCal)	2
Notes: 1. R output: Coefficients: (Intercept) Carbohydrate.Content 73.65 4.65 2. • ⁶ , • ⁷ , • ⁸ and • ⁹ are still available to candidates who have calculated the regression line for carbohydrate content on calories, if the scatter plot in • ¹ , is a scatter plot of carbohydrate on calories.					
Commonly Observed Responses: Candidate A - incorrectly calculates coefficients for carbohydrate content on calories R output: Coefficients: (Intercept) Number.of.Calories -11.6432 0.1993 Leading to: carbohydrate = $-11.6432 + 0.1993 \times \text{calories}$					
	(d)		• ¹⁰ calculate fitted value	• ¹⁰ 348 (kCal)	1
Notes: 1. For • ¹⁰ the number of calories must be stated explicitly. 2. Accept answers from 340 to 360 where candidates have read the value from the regression line.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		• ¹ state critical path	• ¹ A, C, F, G	1
Notes:					
Commonly Observed Responses:					
	(b)		• ² state float time for B	• ² 3 days	1
Notes:					
1. Units must be stated.					
Commonly Observed Responses:					
	(c)		• ³ explain one advantage	• ³ clear and simple to read	1
Notes:					
1. Accept any other reasonable explanation. eg easy to see float times					
2. Do not accept any responses indicating order of precedence, eg easy to see order of activities.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		Method 1 •¹ start to accumulate deposit on 1 April 2021 •² calculate accumulated value of deposit 1 April 2021 •³ calculate accumulated value of deposit on 1 August 2021 •⁴ calculate accumulated value of deposit on 1 April 2022 and final balance	Method 1 •¹ $400 \times 1.012^{\frac{10}{12}} \dots$ •² $400 \times 1.012^{\frac{10}{12}} \times 1.0011^6 = \text{£}406.66 \dots$ •³ $200 \times 1.012^{\frac{6}{12}} \times 1.0011^6 = \text{£}202.53 \dots$ •⁴ $250 \times 1.0011^4 = 251.10 \dots$ Balance = £860.30	4
			Method 2 •¹ start to accumulate balance on 1 August 2021 •² calculate accumulated balance on 1 August 2021 •³ calculate accumulated balance on 1 April 2022 •⁴ calculate final balance 31 July 2022	Method 2 •¹ $400 \times 1.012^{\frac{4}{12}}$ •² $\left(400 \times 1.012^{\frac{4}{12}} \right) + 200 = 601.59$ •³ $\left(601.59 \times 1.012^{\frac{6}{12}} \times 1.0011^2 \right) + 250 = 856.52$ •⁴ $(856.52 \times 1.0011^4) = (\text{£})860.29$	
Notes: 1. Final answer must be stated to 2 decimal places, ignore any rounding errors or truncation.					
Commonly Observed Responses:					
	(b)		•⁵ know how to calculate the present value of the £1000 on 1 August 2022 •⁶ calculate value of deposit on 1 August 2022	•⁵ $1000 \div 1.017^{\frac{4}{12}}$ •⁶ $(\text{£}994.40 - \text{£}860.30) = \text{£}134.10$	2
Notes: 1. Answer must be consistent with previous working.					
Commonly Observed Responses:					
1. $1000 - 860.30 = 139.70$, $139.70 \div 1.017^{\frac{4}{12}} = 138.92$, award • ⁵ only.					

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•¹ complete cells C7, C8, C9 •² use appropriate formula in cell C14 •³ extend spreadsheet and state prediction for week 26	•¹ eg 1750, 80% and 300, stated or implied by •² •² eg = INT(C13*\$C\$8+\$C\$9) •³ 1500	3

Notes:

- ² is not available for candidates who have not used the INTEGER function.
- ³ is still available where no INTEGER function has been used and final answer is rounded or truncated to a whole number.
- Evidence for •³ may appear in cell C39.

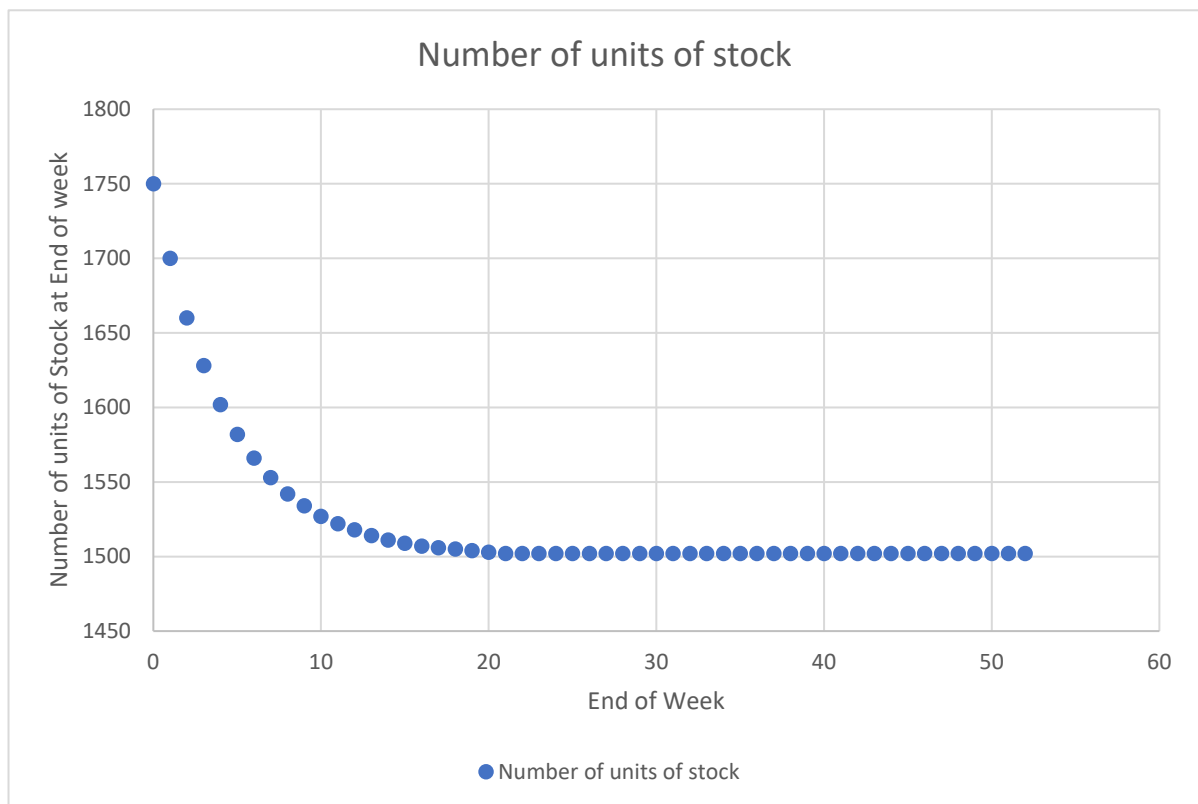
Commonly Observed Responses:

- 1.1501 resulting from not using INTEGER function, award •¹ and •³.

	(b)	(i)	•⁴ extend the spreadsheet to week 52 •⁵ construct appropriate graph	•⁴ evidence of extending spreadsheet •⁵ see notes	2
--	-----	-----	--	--	---

Notes:

- Title and labels are not required to award •⁵.



Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)	(ii)	• ⁶ state appropriate mathematical model	• ⁶ exponential (decay) or recurrence relation	1
Notes:					
Commonly Observed Responses:					
	(c)		• ⁷ comment with justification	• ⁷ no, the building will not have enough space as units of stock are expected to level out at 1500 units of stock	1
Notes:					
1. Answer must be consistent with (b)(ii)					
Commonly Observed Responses:					
For the award of • ⁷ (a) Accept eg • there is not enough space because the levels of stock exceed 1400 • no, because by week 52 there is too much stock • no, because there is not enough space (b) Do not accept eg • any answer with no justification • any answer where (b)(i) is not attempted					
	(d)		• ⁸ appropriate explanation	• ⁸ eg model assumes a fixed rate of stock each week OR eg model assumes a fixed rate of deliveries each week	1
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ explain relative decline in purchasing power between 2015 and	•¹ eg £110.40 in April 2021 would buy the same amount of goods and services as £100 in 2015 OR the increase in the CPI means that I can buy fewer goods with my money in 2021 than in 2015 OR purchasing power has decreased	1
Notes: 1. Answer must reference purchasing, see COR 1.					
Commonly Observed Responses: 1. Can buy less now for the same amount of money, award • ¹ . 2. Purchasing power is 90.58% compared to 2015, award • ¹ . 3. Can buy less now, do not award • ¹ . 4. Prices have risen by 9.42% between April 2015 and April 2021, do not award • ¹ .					
	(b)		•² calculate price of 3-door car in 2015	•² (£)12,778.99	1
Notes: 1. Accept answers that round to (£)12,779 without working.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
9.	(c)		Method 1 • ³ calculate relative rate of inflation • ⁴ calculate National Living Wage (NLW) in line with inflation and state conclusion	Method 1 • ³ 7.78%... stated or implied by • ⁴ • ⁴ £9.60, so NLW rate increase was below inflation	2
			Method 2 • ³ calculate relative rate of inflation • ⁴ calculate percentage increase in NLW and state conclusion	Method 2 • ³ 7.78% ...stated or implied by • ⁴ • ⁴ 6.6% or 106.6% so NLW rate increase was below inflation	

Notes:

1. Accept decimal equivalents.

Commonly Observed Responses:

Candidate A

$$\frac{119.0}{110.4} = 1.077... = 107.8\%$$

$$\frac{9.50}{8.91} = 1.066... = 106.6\%$$

therefore, the increase is not in line, award •³ and •⁴

Candidate B

$$\frac{119 - 110.4}{110.4} = 0.077...$$

$$\frac{9.50 - 8.91}{8.91} = 0.066...$$

therefore the increase is not in line, award •³ and •⁴

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)	(i)	• ¹ calculate CO ₂ emissions	• ¹ 1 178 571 (tonnes a year)	1
Notes: 1. Accept calculations where tonnes per car is rounded or truncated to 2 significant figures. eg tonnes per car is 2.1, leading to 1,155,000 (tonnes a year). 2. Accept rounded answers where appropriate working is shown.					
Commonly Observed Responses:					
		(ii)	• ² state appropriate reason	• ² eg newspaper bias	1
Notes: 1. Accept any relevant comment about the source of the article or the information contained in the article. 2. Do not accept comments that state explicitly that the information in the article is wrong. 3. For the award of • ² (a) Accept eg • unconfirmed source • cars or fuel may change • other factors may effect CO₂ emissions • information in the article may not be accurate (b) Do not accept eg • information in the article is not accurate					
Commonly Observed Responses:					
	(b)	(i)	• ² state independent variable	• ² (additional) load or <i>m</i>	1
Notes:					
Commonly Observed Responses:					
		(ii)	• ³ calculate fuel economy	• ³ 68.8... (mpg)	1
Notes:					
Commonly Observed Responses:					
	(c)		• ⁴ correct graph • ⁵ give reason	• ⁴ graph A • ⁵ eg depth increases faster at the beginning/slows in the middle/faster at the end	2
Notes: 1. Explanation must describe the change in depth.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)	(i)	•¹ calculate monthly interest rate •² create formulae for interest repayment, capital repayment and loan outstanding •³ complete remainder of loan schedule for 36 months •⁴ calculate monthly repayment and adjust final repayment	•¹ 0.51% •² D17, E17 and F17 (see spreadsheet) •³ check cells D52, E52, F52 (see spreadsheet) •⁴ (£) 182.87, (£) 182.94 OR (£) 182.88, (£) 182.58	4

Notes:

1. For marks •¹, •² and •³ see below

	Formula view	Value view
• ¹	$=(1+C7)^{(1/12)}-1$	0.51%
• ²	D17=ROUND(\$C\$8*F16, 2), E17=C17-D17, F17=F16-E17	(£) 30.63, (£) 152.24, (£) 5847.76
• ³	D52=ROUND(\$C\$8*F51, 2), E52=C52-D52 F52=F51-E52	(£) 0.93, (£) 182.01, (£) 0

Commonly Observed Responses:

Candidate A - based on candidate using 0.525%

For an incorrect monthly interest rate, mark •¹ is not available but follow through working to award marks •², •³ and •⁴.

£183.35, £183.28

OR

£183.34, £183.64

		(ii)	• ⁵ calculate total interest paid	• ⁵ (£) 583.39	1
--	--	------	--	---------------------------	---

Notes:

1. Answer must be consistent with previous working.

Commonly Observed Responses:

	(b)		• ⁶ state valid reason	• ⁶ eg poor credit rating, affordability etc	1
--	-----	--	-----------------------------------	---	---

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.	(c)	(i)	•⁷ complete ‘Loan company’ worksheet using dummy monthly interest rate •⁸ determine annual effective rate of interest	•⁷ D18, E18, F18 (see spreadsheet) •⁸ 32.61...%	2
Notes:					
		Formula view		Value view	
• ⁷		D18=ROUND(\$C\$8*F17,2), E18=C18-D18, F18=F17-E18		(£) 142.80, (£) 107.20, (£) 5892.80	
Commonly Observed Responses:					
		(ii)	•⁹ determine difference in total interest paid	•⁹ (£) 2416.61 OR (£) 2416.62	1
Notes:					
1. Answer must be consistent with previous working					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]