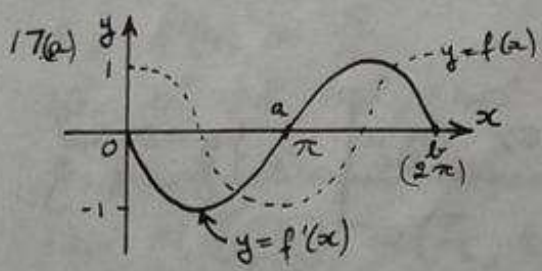


S.C.E. REVISED HIGHER - ANSWERS

1995 I

1. 4 units. 2. (a) Verify that $f(3) = 0 \therefore (x-3)$ is a factor of $f(x)$.
 (b) $(x-3)(x+4)(2x+1)$
3. $2x^3 - \frac{1}{2}x^2 + \sin x + C$ 4. $k = 3$ 5. $4x + 3y = 1$
6. Prove that A, A_2 has equation $3y = 2x + 5$. Then verify that $(5, 4)$ does not satisfy this equation \therefore ACHILLES DOES NOT PASS OVER THE SUBMARINE.
 also, prove that B, B_2 has equation $4y = 5x - 9$. Then verify that $(5, 4)$ DOES satisfy this equation \therefore BELLIGERENT DOES PASS OVER THE SUBMARINE.
7. $-\frac{8}{x^3} + \frac{3}{2}x^{1/2}$ 8. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ 9. $(x-1)^2 + (y-2)^2 = 18$
10. (a) $b = -10$ (b) Prove that $f'(-2) = 33$. Since $f'(-2) > 0$, f is increasing at P
11. (a) $\frac{x^2-2}{x^2-4}$ (b) $\{x : x \in \mathbb{R}, x \neq 2 \text{ or } -2\}$ 12. $\sin 2\alpha = \frac{3\sqrt{11}}{10}$
13. $k = \sqrt{29}$, $x = 68.2^\circ$ 14. $f'(x) = \text{gradient of line} = -\frac{1}{3}$
15. $x = 60^\circ, 180^\circ, 300^\circ$ 16. 9



- (b) $f(x) = \cos x$
 $f'(x) = -\sin x$ 19.8.
18. (a) $k = 0.035$ (b) $t = 8.6$ minutes
19. [replace $(3,0)$ and $(4,3)$ into equation]
 $a = 3, b = 2$

20. $k = -5$ or 3
21. (a) Speed = $20 - 10t$ (b) 0 m/s. Ball has reached its maximum height.
 Speed when thrown = 20 m/s

1995 II

1. (a) Verify that $AB = AC = 3\sqrt{5} \therefore \Delta ABC$ is isosceles.
 (b) (i) [AD: $x = 4$, BE: $2y - x = 3$] $\therefore H(4, 3\frac{1}{2})$
 (ii) Check that $HD = 1\frac{1}{2}$ units, $DA = 6$ units $\therefore H$ is $\frac{1}{4}$ the way up DA
2. (a) — (b) Tangent meets curve AGAIN at $(-2, -18)$
3. KILLPEST $[u_{n+1} = 0.35u_n + 500]$ \rightarrow 769 pests in the long term
 PESTKILL $[u_{n+1} = 0.15u_n + 650]$ \rightarrow 765 pests in the long term
 \therefore PESTKILL is more effective in the long term
4. (a) (i) $f(x) = 2\sin 3x \therefore a = 3, b = 2$ (ii) $g(x) = 3\cos 3x \therefore c = 3, d = 3$
 (b) [let $h(x) = 2\sin 3x + 3\cos 3x = q \sin(3x + r)$
 $\therefore q = \sqrt{13}, r = 56.3^\circ \therefore h(x) = \sqrt{13} \sin(3x + 56.3^\circ)$

$\left[\vec{QP} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}, \vec{QR} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \right] \rightarrow \angle PQR = 60^\circ.$

(b)(i) $T(2\frac{1}{3}, 3\frac{1}{3}, 1\frac{1}{3})$ (ii) Verify that $PT = QT = RT = \frac{2}{3}\sqrt{6}$
 $\therefore P, Q, R$ are equidistant from T .

(c)(i) Verify that $PA = QA = RA = \sqrt{3}$. $\therefore P, Q, R$ are equidistant from A .

(ii) By construction, T lies within $\triangle PQR$ - ie T is coplanar with P, Q, R .

also $PA[\sqrt{3}] > PT[\frac{2}{3}\sqrt{6}] \therefore A$ is not coplanar with P, Q, R .

$\therefore T$ is the centre of the circle passing through P, Q, R (AND NOT A).

6. [THE GAUSSIAN ELIMINATION METHOD AS DESCRIBED MUST BE USED.]

[1st obtain $\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \\ 0 & 0 & 2 & 2 \end{array} \therefore x = 3, y = -1, z = 1$]

7. $a = -\frac{1}{3}, b = \frac{1}{3}, c = 3$

8. [YOU MUST OBTAIN: $A(-12, -15) r_1 = 5$ $C(24, 12) r_3 = 10$
 $AC = 45$ units
 $r_2 = 15$ (radius of central circle) $B(4, -3)$ [QUITE HARD]]

$(x-4)^2 + (y+3)^2 = 225$

9. (a) - (b) $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x + C$

10. (a) 48 units^2 (b) -

(c)(i) - (ii) Other 2 solutions are $6+6\sqrt{3}, 6-6\sqrt{3}$ (or $16.39, -4.39$ APPROX.)

(iii) k must lie between 0 and 12 which the other 2 solutions do NOT satisfy.
 $\therefore k = 6$ is the only valid solution to this problem.

11. (a) - (b) -

(c) $x = 4$ (not -4 since A is in the 1st quadrant)

[Table of values required to verify MAX.]

$\therefore \text{MAX. LENGTH} = \underline{10 \text{ units}}$.