

S.C.E. HIGHER ANSWERS - 1996

PAPER I

1. $3x + 2y = 17$

2. $a = 2\frac{1}{2}$

3. $(\frac{5\pi}{6}, 3)$

4. $2x + y = 10$

5. 9

6. Verify that $\vec{AB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$

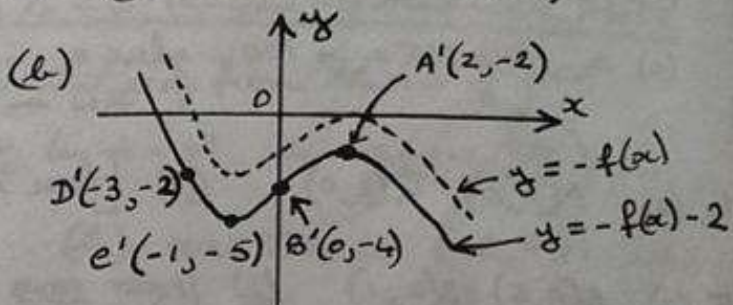
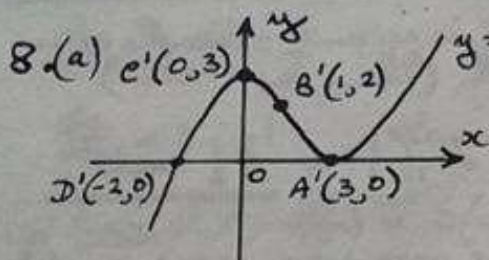
7. $x(x-1)(x^2+x+1)$

$\therefore \vec{AB} = \frac{2}{3} \vec{BC}$

$\therefore AB$ is parallel to BC

$\therefore A, B, C$ are collinear [$\because B$ is a common point]

B divides AC in the ratio $2:3$ ($\therefore AB:BC = 2:3$)



9. $f'(4) = \frac{5}{16}$

10. $0^\circ, 120^\circ, 180^\circ, 240^\circ$ [NB NOT 360° since $0^\circ \leq x < 360^\circ$]

11. (a) Limit exists because 0.3 lies between -1 and 1 (b) $7\frac{1}{7}$

12. $P(\frac{\pi}{6}, \frac{1}{2})$ $Q(\frac{\pi}{2}, \frac{1}{2})$

13. $\frac{dy}{dx} = \frac{-\sin x}{2\sqrt{1+\cos x}}$

14. Take 2 lines $\begin{cases} 2x + 3y = 4 \\ 3x - y = 17 \end{cases}$. Verify that they intersect at $(5, -2)$. Then verify that $(5, -2)$ does NOT lie on the 3rd line $x - 3y - 10 = 0$. \therefore 3 LINES ARE NOT CONCURRENT.

15. [Verify that $BD = 5$ and $AD = 2\sqrt{6}$; then use the expansion of $\cos(x+y)$]

16. f is increasing when $x < -2$ AND $x > 3$ [TABLE OF VALUES REQUIRED]

17. $4(x+1)^2 - 9$

18. $\sin x = \frac{4\sqrt{5}}{9}$

19. (a) $k = 0.019$ (b) [FALLS TO 56.56°C IN NEXT 15 MINS] \therefore IT FALLS 18.44°C IN NEXT 15 MINS

20. $(x-3)^2 + (y-4)^2 = 25$.

PAPER II

1. (a) S.P.'s are $(0, 3)$ AND $(3, -24)$ (b) $(0, 3)$ is a point of inflection $(3, -24)$ is a MINIMUM S.P.

2. (a) Verify that $m_{AB} m_{BC} = -1$ WITH A CLEAR CONCLUSION $\begin{pmatrix} m_{AB} = 2 \\ m_{BC} = -\frac{1}{2} \end{pmatrix}$

(b)(i) $AD: x - 3y = 6$

(ii) $M(1, -\frac{5}{3})$

$BE: 4x + 3y = -1$

(a) $Q(2, 2, 9)$ $R(21, 3, 12)$ (b) $\hat{QPR} = 83.4^\circ$
 (a) (i) $g[f(x)] = 4x^2 + 4x + 1 + k$ (ii) $f[g(x)] = 2x^2 + 2k + 1$
 (b) (i) [ANSWER IN QUESTION] (ii) $\Delta = 64$ \therefore Roots are REAL AND DISTINCT
 (iii) $k = -2$

5. [Area I = $\frac{1}{2} - \frac{\sqrt{3}}{4}$; Area II = $\frac{1}{2}$] \therefore TOTAL AREA = $1 - \frac{\sqrt{3}}{4}$ units²

6. (a) Line l: $y = 2x$ Parabola k: $y = 2x^2$ Circle c: $x^2 + y^2 = 5$
 (b) l' : $y = 2x - 4$ k' : $y = -2x^2$ c' : $(x-2)^2 + y^2 = 5$
 (c) $(-2, -8)$ AND $(1, -2)$

7. (a) $\sqrt{13} \cos(x - 56.3^\circ)$ (b) $138.3^\circ, 334.3^\circ$

8. (a) $5x + y = -3$; $B(-1, 2)$
 (b) $\frac{4}{3}$ units²

9. (a) $Y = 3X + 0.7$
 (b) [FIRST REPLACE Y BY $\log_e y$ AND X BY $\log_e x$] $\therefore \begin{cases} a = 2.014 \\ b = 3 \end{cases}$
 TO OBTAIN $\therefore y = 2.014x^3$

10. (a) By symmetry, result needs only to be proved for one point
 at $(3, 7)$: PROVE THAT TANGENT TO PARABOLA HAS GRADIENT $\frac{2}{3}$
 AND PROVE THAT TANGENT TO CIRCLE HAS GRADIENT $-\frac{3}{2}$.
 Then conclude that since product of gradients of tangents is -1 , the
 tangents are perpendicular. \therefore CURVES ARE ORTHOGONAL.

(b) (i) $2y = 3x + 23$
 (ii) $x^2 + (y - 11\frac{1}{2})^2 = 29\frac{1}{4}$

11. (a) (i) $h = 5 - x - \frac{1}{2}\pi x$
 (ii) [START WITH $L = 2(\text{area of rectangle}) + 1(\text{area of } \frac{1}{2} \text{ circle})$]

(b) $x = \frac{20}{8 + 3\pi}$ ($\doteq 1.1478$)
 $h = \frac{20 + 5\pi}{8 + 3\pi}$ ($\doteq 2.0493$)

NB: TABLE OF VALUES IS REQUIRED