1997 S.C.E. HIGHER I (ANSWERS)

1. $2x + y = -3$

2. Verify that $\vec{AB} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} 2 \\ -8 \\ 4 \end{pmatrix}$

$\therefore \vec{AB} = \frac{1}{2}\vec{BC}$

$\therefore$ AB is parallel to BC

$\therefore$ A, B, C are collinear ($\because$ of the common point B)

3. $f[g(x)] = 2(\sin x + \cos x)$

$g[f(x)] = \sin 2x + \cos 2x$

4. (a) $\vec{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$ (b) $PQ = 9$ units

5. (a) $\underline{a}$ real root is $x = 2$ (b) Other roots come from $2x^2 + x + 4 = 0$

$\therefore \Delta = -31$

Since $\Delta < 0$, there are no other real roots

6. $x = -1$ 7. [You will need to construct a Δ from $\tan x = \frac{4}{3}$].

8. $\frac{dy}{dx} = 4x + 1$; then replace $\frac{dy}{dx}$ by $4x+1$ in $x\left(1 + \frac{dy}{dx}\right)$.

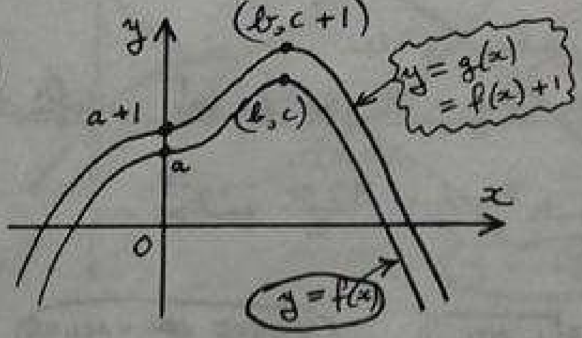
9. (a) Prove that $f(x) = 2(x+2)^2 - 11$ (b) MIN T.P. is $(-2, -11)$

10. $\frac{14}{3}$ 11. $\sqrt{29}\sin(x - 68.2^\circ)$ 12. $(x-13)^2 + (y-4)^2 = 17$

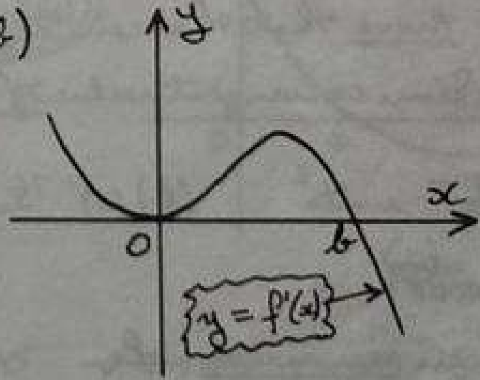
13. $\underline{a}\cdot(\underline{b} + \underline{c}) = 0$; $\underline{a}$ is perpendicular to $\underline{b} + \underline{c}$.

14. $c < 13$ 15. $f(x) = \frac{1}{2}\sin 2x + \frac{3}{4}$

16. (a)



(b)



(c) $g'(x)$ is the same graph as $f'(x)$

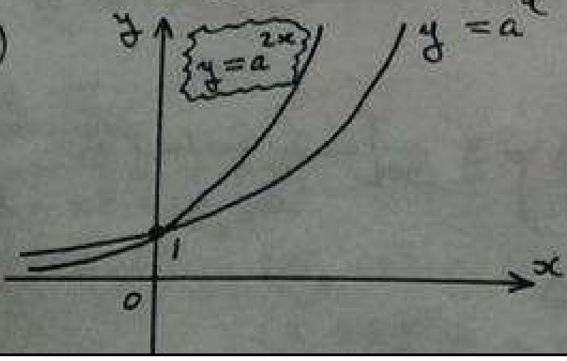
17. $A(-4\frac{1}{2}, 0)$ $B(14.91, 8)$

18. (a) [Replace $\cos 2x$ by $1 - 2\sin^2 x$ and $\cos^2 x$ by $1 - \sin^2 x$]

(b) $x = 19.45^\circ, 160.55^\circ, 270^\circ$

19. (a) $t = a$; $u = 0$ (c) Point of intersection is $(1, a^2)$

(b)



20. $a = 51.34^\circ$.

HIGHER II

1. (a) $A(1,3)$ $B(-3,-5)$ (b)(i) $C(-5,1)$ (ii) $x+2y=-3$

2. (a) $R(7,-1,6)$ (b) $\left[\vec{SR}=\begin{pmatrix}9\\-3\\1\end{pmatrix}\ \vec{SP}=\begin{pmatrix}1\\1\\-3\end{pmatrix}\right]$ $P\hat{S}R = 84{\cdot}56^\circ$

3. (a) $\left[\begin{array}{l} u_0(1^{ST}\text{JAN})=1000 \quad u_1(1^{ST}\text{FEB})=1105 \quad u_2(1^{ST}\text{MAR})=1210{\cdot}53 \\ u_3(1^{ST}\text{APRIL})=1316{\cdot}58 \quad u_4(1^{ST}\text{MAY})=1423{\cdot}16 \quad u_5(1^{ST}\text{JUNE})=1530{\cdot}28 \quad u_6(1^{ST}\text{JULY})=1537{\cdot}93\end{array}\right]$

∴ Amount on JUNE 30^{TH} = **£1537·93**

(b) [Continue until $u_{10}(1^{ST}\text{NOV}) = 2073{\cdot}94$] ∴ First exceeds £2000 on 1^{ST} NOVEMBER

(c) $u_{n+1} = 1{\cdot}005\,u_n + 100$ where u_n is the amount in account n months after 1^{ST} JAN on 1^{ST} of month (where $u_0 = 1000$).

4. (a) $P(1,0)$ $Q(2,0)$ (b) $\left[\text{TOTAL AREA} = 1\tfrac{11}{12} + \tfrac{7}{12}\right] = 2\tfrac{1}{2}$ units^2

5. (a) $A(0,5)$ $B(2,1)$ (b) [FIRST PROVE THAT CURVES MEET WHEN $x=0$ or 4]

∴ AREA ENCLOSED = $21\tfrac{1}{3}$ units^2

(c) $m=10$, $n=-1$

6. (a) $m_{TGT} = -\dfrac{1}{a^2}$ (b) — (c)(i) [PROVE THAT B IS $(0,\tfrac{2}{a})$ AND C IS $(2a,0)$]

∴ AREA OF ΔOBC = 2 units^2

(ii) area of Δ OBC is always 2 units^2 regardless of position of tangent at A

7. $\dfrac{5x+1}{(x-4)(x+3)} = \dfrac{3}{x-4} + \dfrac{2}{x+3}$

8. (a) $k = -0{\cdot}000122$ (b) $88{\cdot}5\%$

9. (a) LIKELIEST : $p=3$ $q=1$ $r=-140^\circ$ $u=230^\circ$

(OTHERS : $p=3$ $q=1$ $r=220^\circ$ $u=230^\circ$
$p=-3$ $q=1$ $r=40^\circ$ $u=230^\circ$
$p=-3$ $q=1$ $r=-320^\circ$ $u=230^\circ$)

(b) $s=-0{\cdot}928$
$t=120.53^\circ$
[USE EQUATION WITH REPLACEMENTS FOR p, r AND q]

10. (a)(i) [USE SIMILAR Δ's ADE AND ABC] (ii) [$V=(2x)(2x)(10-\tfrac{5}{2}x)$ etc]

(b) [PROVE THAT GREATEST VOLUME OCCURS WHEN $x=\tfrac{8}{3}$ (not $x=0$ ∵ $x>0$)]

∴ DIMENSIONS ARE $\tfrac{16}{3}$ cm × $\tfrac{16}{3}$ cm × $\tfrac{10}{3}$ cm

11. (a)(i) — (ii) $OC = 1+2\cos p$ [and finally use $d^2 = OB^2+OC^2$]

(b)(i) $d^2 = 6+4\sqrt{2}\cos(p-\tfrac{\pi}{4})$ (ii) MAX. VALUE OF d^2 IS $6+4\sqrt{2}$; OCCURS WHEN $p=\tfrac{\pi}{4}$.

(c)(i) [REPLACE p by $\tfrac{\pi}{4}$ in $OB=1+2\sin p$]; $BD = \tfrac{1}{2}(\sqrt{2}+2)$

(ii) $\sqrt{6+4\sqrt{2}} = 2+\sqrt{2}$