

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	1.1.8, 1.1.7	CN	C	2001 q1

Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point $(2, -1)$.

3

Give 1 mark for each •

Illustrations for awarding each •

ans: $2x + 3y = 1$

3 marks

- ¹ ss : express in standard form
- ² ic : interpret gradient
- ³ ic : state equation of st line

- ¹ $y = -\frac{2}{3}x + \frac{5}{3}$ stated or implied by •²
- ² $m_{line} = -\frac{2}{3}$ stated or implied by •³
- ³ $y - (-1) = -\frac{2}{3}(x - 2)$

Notes

- 1 •³ is only available for candidates who attempt to find or state the gradient.
- 2 •³ is still available even though •¹ and •² may not have been awarded.

example for note 2

$$m = 2$$

$$y - (-1) = 2(x - 2) \text{ earns } \bullet^3$$

example for note 1

$$y - (-1) = 7(x - 2) \text{ earns no marks.}$$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	2.1.7	CN	C	2001 q2

For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots?

3

Give 1 mark for each •

Illustrations for awarding each •

ans: $k = \frac{1}{4}$

3 marks

- ¹ ss : know to set disc. to zero
- ² ic : substitute a, b and c into discriminant
- ³ pd : process equation in k

- ¹ $b^2 - 4ac = 0$ stated or implied by •²
- ² $(-5)^2 - 4 \times (k + 6)$
- ³ $k = \frac{1}{4}$

Notes

- 1 “= 0” must occur on one of the lines in the solution to this question.
- 2 If the expression for the discriminant involves x , no marks are available.
- 3 If the phrase “discriminant = 0” appears at the start, then •¹ can only be awarded if •² is awarded.

ie $disc = 0$

$$25 - 4(k + 6) = 0$$

$$k = \frac{1}{4}$$

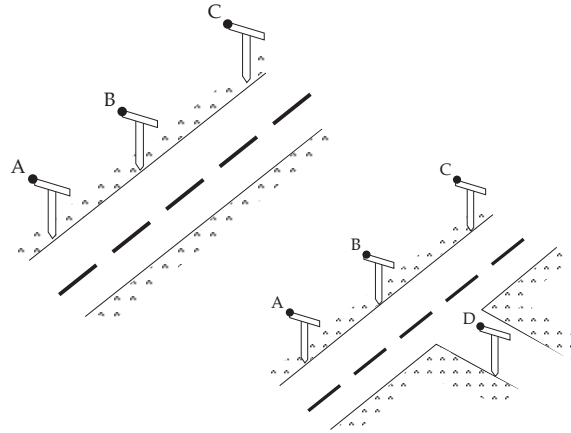
can be awarded 3 marks.

Alternative sol

- ¹ $x^2 - 5x + k + 6 = 0$
- ² $x^2 - 5x + \left(\frac{5}{2}\right)^2 = 0$ has equal roots
 $k + 6 = \frac{25}{4}$
- ³ $k = \frac{1}{4}$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	3	3.1.7	CN	C	2001 q3
b)	3	3.1.10	CN	C	

(a) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points $A(-8, -10, -2)$, $B(-2, -1, 1)$ and $C(6, 11, 5)$. Determine whether or not the section of road ABC has been built in a straight line.



(b) A further T-rod is placed such that D has coordinates $(1, -4, 4)$. Show that DB is perpendicular to AB.

3
3

Give 1 mark for each •

Illustrations for awarding each •

a ans: the road ABC is straight 3 marks

- ¹ ic : interpret vector (eg \vec{AB})
- ² ic: interpret multiple of vector
- ³ ic : complete proof

•¹ e.g. $\vec{AB} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$

•² e.g. $\vec{BC} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix} = \frac{4}{3} \vec{AB}$

- ³ a) a common direction exists
and b) a common point exists
so A, B, C collinear

Notes

- 1 For •³, accept references to “they” are parallel.
- 2 For (b) Converse of Pythagoras provides an alternative.[See below].
- 3 Other methods include using the cosine rule and the scalar product.

- ¹ calculating $AB^2 = 126, BD^2 = 27, AD^2 = 153$
- ² stating $\cos \hat{A}BD = \frac{126+27-153}{\sqrt{\dots}\sqrt{\dots}}$
- ³ $= 0$ so $\angle ABD = 90^\circ$

- ¹ calculating $AB^2 = 126, BD^2 = 27, AD^2 = 153$
- ² stating $126 + 27 = 153$
- ³ by converse of Pythagoras $\angle ABD = 90^\circ$

b ans: proof 3 marks

- ⁴ ic : interpret vector (ie \vec{BD})
- ⁵ ss: state requirement for perpend.
- ⁶ ic : complete proof

•⁴ $\vec{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

•⁵ $\vec{AB} \cdot \vec{BD} = 0$

•⁶ $\vec{AB} \cdot \vec{BD} = 18 - 27 + 9 = 0$

•⁴ $\vec{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

•⁵ $\vec{AB} \cdot \vec{BD} = 18 - 27 + 9$

•⁶ $= 0$ so AB is at right angles to BD

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	2	1.2.8	NC	C	2001 q4

Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$.

2

Give 1 mark for each •

Illustrations for awarding each •

ans: $(x + 1)^2 - 9$

2 marks

•¹ ss : eg start to complete square•² pd : complete process•¹ $(x + 1)^2 \dots\dots$ •² $(x + 1)^2 - 9$

OR

•¹ $a = 1$ •² $b = 9$

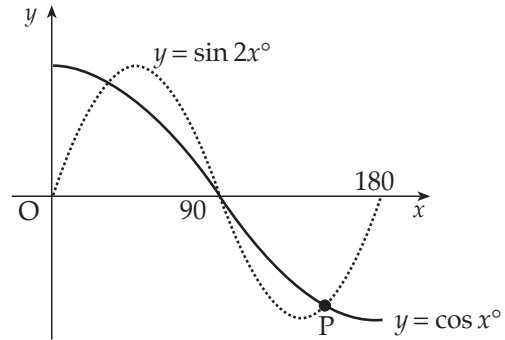
OR

•¹ $x^2 + 2x - 8 \equiv x^2 + 2ax + a^2 - b$ •² $a = 1$ and $b = 9$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	4,1	2.3.5	NC	C	2001 q5

5 (a) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$.

(b) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$. Use your solutions in (a) to write down the coordinates of the point P.



4

1

Give 1 mark for each •

Illustrations for awarding each •

a ans: 30, 90, 150 4 marks

- ¹ ss : use double angle formula
- ² pd : factorise
- ³ pd : process
- ⁴ pd : process

- ¹ $2 \sin x^\circ \cos x^\circ$
 - ² $\cos x^\circ(2 \sin x^\circ - 1)$
 - ³ $\cos x^\circ = 0, \sin x^\circ = \frac{1}{2}$
 - ⁴ 90, 30, 150
-
- ³ $\sin x^\circ = \frac{1}{2}$ and $x = 30, 150$
 - ⁴ $\cos x^\circ = 0$ and $x = 90$

Notes

1 The inclusion of wrong answer(s) means the mark is not awarded (•⁴ in method 1, •³ or •⁴ in method 2).

b ans: $\left(150, -\frac{\sqrt{3}}{2}\right)$ 1 mark

- ⁵ ic : interpret graph

- ⁵ $\left(150, -\frac{\sqrt{3}}{2}\right)$

Notes

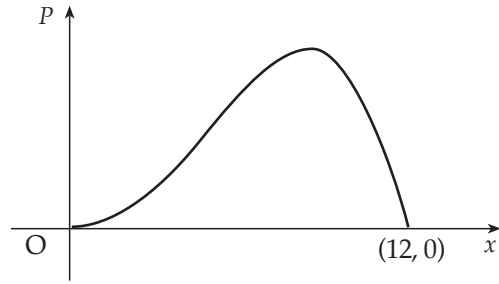
2 Accept $y = \cos 150^\circ = -\frac{\sqrt{3}}{2}$ as poor form

3 Wrong formula:

- ¹ X $2 \cos^2 x^\circ - 1$
- ² $\sqrt{(2 \cos x^\circ + 1)(\cos x^\circ - 1)} = 0$
- ³ $\sqrt{\cos x^\circ = -\frac{1}{2}, \cos x^\circ = 1}$
- ⁴ $\sqrt{0, 120, (240, 360)}$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	5	1.3.15	NC	C	2001 q6

A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \leq x \leq 12$. Find the value of x which gives the maximum profit.



5

Give 1 mark for each •

Illustrations for awarding each •

ans: $x = 9$

5 marks

- ¹ ss : start diff. process
- ² pd : process
- ³ ss : set derivative to zero
- ⁴ pc : process
- ⁵ ic : interpret solutions

- ¹ $\frac{dP}{dx} = 36x^2 \dots\dots$ **or** $\frac{dP}{dx} = \dots\dots - 4x^3$
- ² $\frac{dP}{dx} = 36x^2 - 4x^3$
- ³ $\frac{dP}{dx} = 0$
- ⁴ $x = 0$ **and** $x = 9$
- ⁵ nature table about $x = 9$ **and** $x = 9$

Notes

- 1 The “= 0” shown in •³ may appear anywhere in the working but must appear explicitly.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	2	1.2.6	NC	C	2001 q7
b)	5	2.3.5	NC	C	

Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for

(i) $f(h(x))$

(ii) $g(h(x))$.

2

(b) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation

$$f(h(x)) - g(h(x)) = 1 \text{ for } 0 \leq x \leq 2\pi.$$

5

Give 1 mark for each •

Illustrations for awarding each •

a ans: $\sin(x + \frac{\pi}{4})$, $\cos(x + \frac{\pi}{4})$ 2 marks

•¹ ic : interpret composite functions

•² ic : interpret composite functions

•¹ $\sin(x + \frac{\pi}{4})$

•² $\cos(x + \frac{\pi}{4})$

Notes

1 One mark may be awarded for

“ $f(x + \frac{\pi}{4})$ and $g(x + \frac{\pi}{4})$ ”

2 For $\sin x + \frac{\pi}{4}$ and $\cos x + \frac{\pi}{4}$ award 1 mark only, unless there is evidence in part b that they have been expanded correctly, in which case treat as bad form and award 2 marks.

3 Do not penalise the appearance of 45°

b ans: proof and $x = \frac{\pi}{4}, \frac{3\pi}{4}$ 5 marks

•³ ss : expand $\sin(x + \frac{\pi}{4})$

•⁴ ic : interpret

•⁵ ic : substitute

•⁶ pd : start solving process

•⁷ pd : process

•³ $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ and complete

•⁴ $g(h(x)) = \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x$

•⁵ $(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x) - (\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x)$

•⁶ $\frac{2}{\sqrt{2}} \sin x$

•⁷ $x = \frac{\pi}{4}, \frac{3\pi}{4}$

Notes

4 If the evidence for •⁵ has no brackets, •⁵ can only be awarded if there is evidence further on that brackets has been implied.

5 •⁷ is only available for answers in radians.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	3.3.3, 3.3.4	NC	C	2001 q8

Find x if $4\log_x 6 - 2\log_x 4 = 1$.

3

Give 1 mark for each •

Illustrations for awarding each •

ans: 81

3 marks

- ¹ pd : use log-to-index rule
- ² pd : use log-to-division rule
- ³ ic : interpret base for $\log_x a = 1$ and simplify

- ¹ $\log_x 6^4 - \log_x 4^2$
- ² $\log_x \frac{6^4}{4^2}$
- ³ all processing leading to $x = 81$

Further illustrations

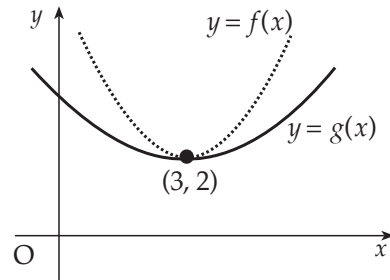
- ¹ $4\log_x 6 - 4\log_x 2 = 1$
 $4(\log_x 6 - \log_x 2) = 1$
- ² $4\log_x \frac{6}{2} = 1$
 $\log_x \frac{6}{2} = \frac{1}{4}$
 $x^{\frac{1}{4}} = 3$
- ³ $x = 81$

- ¹ $4\log_x 6 - 4\log_x 2 = 1$
 $4(\log_x 6 - \log_x 2) = 1$
- ² $4\log_x \frac{6}{2} = 1$
 $\log_x \left(\frac{6}{2}\right)^4 = 1$
- ³ $x = 81$

- ¹ $2\log_x 6 - \log_x 4 = \frac{1}{2}$
 $\log_x 6^2 - \log_x 4 = \frac{1}{2}$
- ² $\log_x \frac{6^2}{4} = \frac{1}{2}$
 $x^{\frac{1}{2}} = 9$
- ³ $x = 81$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	2	1.2.4, 1.3.8	CN	C	2001 q9

The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at $(3, 2)$. Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$.



2

Give 1 mark for each •

Illustrations for awarding each •

ans: for all k

2 marks

- ¹ ss : use $\frac{d}{dx}$ ("quadratic") = "linear"
- ² ic : interpret stationary point

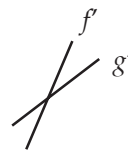
- ¹ st line for f' thr' $(3, 0)$, $m_{f'} > 0$
- ² st line for g' thr' $(3, 0)$, $m_{f'} > m_{g'} > 0$



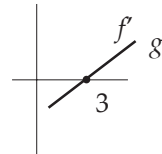
- ¹ st lines for f' and g' , with $m_{f'} > m_{g'} > 0$
- ² two lines intersecting at $(3, 0)$

Notes

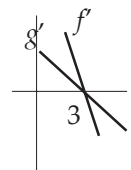
- 1 Award 0 marks for two curves through anywhere.
- 2 Further illustrations:



method 2 •¹



method 1 •¹



method 2 •²

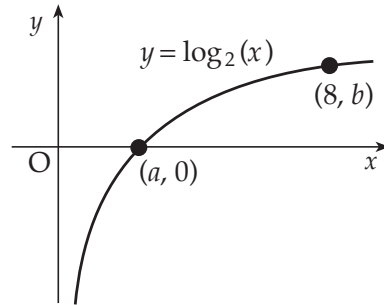
part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	1	1.2.5	CN	B	2001 q10
b)	3	1.2.4, (3.0.0HG?)	CN	A	

10 The diagram shows a sketch of part of the graph of

$$y = \log_2(x).$$

(a) State the values of a and b .

(b) Sketch the graph of $y = \log_2(x + 1) - 3$.



1

3

Give 1 mark for each •

Illustrations for awarding each •

a ans: $a = 1, b = 3$

1 mark

•¹ $a = 1$ and $b = 3$

•¹ pd : use $\log_p q = 0 \Rightarrow q = 1$ and evaluate $\log_p p^k$.

b ans: sketch

3 marks

•² a "log - shaped" graph of the same orientation

•² ss : use a translation

•³ sketch passes through $(0, -3)$ (labelled)

•³ ic : identify one point

•⁴ sketch passes through $(7, 0)$ (labelled)

•⁴ ic : identify a second point

Notes

1 Do not penalise any errors made in relation to the asymptote, missing or otherwise.

2 $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ is the correct translation!. You may also

consider $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$. •² is still available

and in addition one mark (from •³ and •⁴) may be awarded for both points consistent with the wrong translation.

Do not consider any other translation.

