

**X056/301**

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NATIONAL  
QUALIFICATIONS  
2001

THURSDAY, 17 MAY  
9.00 AM - 10.10 AM

MATHEMATICS  
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

**Read Carefully**

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

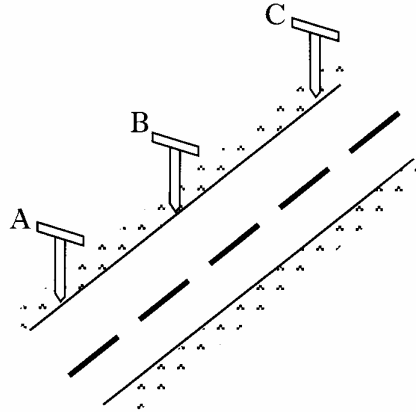
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

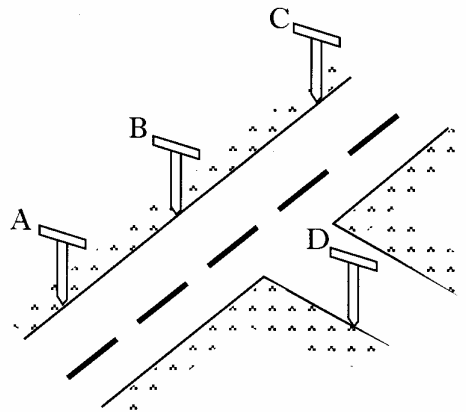
1. Find the equation of the straight line which is parallel to the line with equation  $2x + 3y = 5$  and which passes through the point  $(2, -1)$ . 3
2. For what value of  $k$  does the equation  $x^2 - 5x + (k + 6) = 0$  have equal roots? 3

3. (a) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points  $A(-8, -10, -2)$ ,  $B(-2, -1, 1)$  and  $C(6, 11, 5)$ .  
Determine whether or not the section of road ABC has been built in a straight line.



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- (b) A further T-rod is placed such that D has coordinates  $(1, -4, 4)$ .  
Show that DB is perpendicular to AB.



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4. Given  $f(x) = x^2 + 2x - 8$ , express  $f(x)$  in the form  $(x + a)^2 - b$ . 2

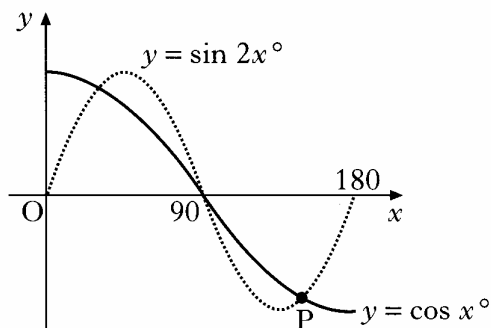
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5. (a) Solve the equation  $\sin 2x^\circ - \cos x^\circ = 0$  in the interval  $0 \leq x \leq 180$ .

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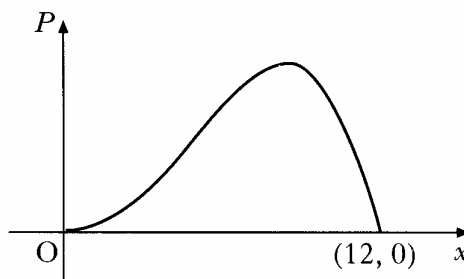
- (b) The diagram shows parts of two trigonometric graphs,  $y = \sin 2x^\circ$  and  $y = \cos x^\circ$ .

Use your solutions in (a) to write down the coordinates of the point P.



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6. A company spends  $x$  thousand pounds a year on advertising and this results in a profit of  $P$  thousand pounds. A mathematical model, illustrated in the diagram, suggests that  $P$  and  $x$  are related by  $P = 12x^3 - x^4$  for  $0 \leq x \leq 12$ . Find the value of  $x$  which gives the maximum profit.



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7. Functions  $f(x) = \sin x$ ,  $g(x) = \cos x$  and  $h(x) = x + \frac{\pi}{4}$  are defined on a suitable set of real numbers.

(a) Find expressions for:

- (i)  $f(h(x))$ ;  
 (ii)  $g(h(x))$ .

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(b) (i) Show that  $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$ .

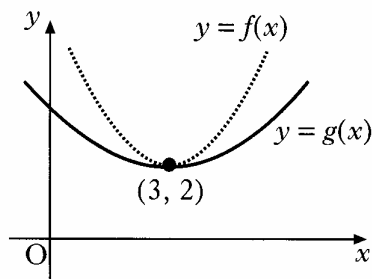
(ii) Find a similar expression for  $g(h(x))$  and hence solve the equation  $f(h(x)) - g(h(x)) = 1$  for  $0 \leq x \leq 2\pi$ .

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8. Find  $x$  if  $4 \log_x 6 - 2 \log_x 4 = 1$ .

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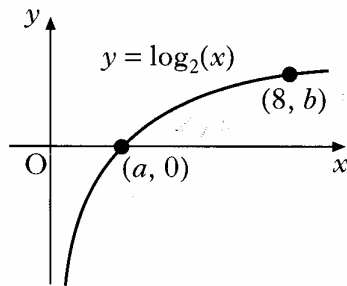
9. The diagram shows the graphs of two quadratic functions  $y = f(x)$  and  $y = g(x)$ . Both graphs have a minimum turning point at  $(3, 2)$ .



Sketch the graph of  $y = f'(x)$  and on the same diagram sketch the graph of  $y = g'(x)$ .

2

10. The diagram shows a sketch of part of the graph of  $y = \log_2(x)$ .



(a) State the values of  $a$  and  $b$ .

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(b) Sketch the graph of  $y = \log_2(x + 1) - 3$ .

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11. Circle P has equation  $x^2 + y^2 - 8x - 10y + 9 = 0$ . Circle Q has centre  $(-2, -1)$  and radius  $2\sqrt{2}$ .

(a) (i) Show that the radius of circle P is  $4\sqrt{2}$ .

(ii) Hence show that circles P and Q touch.

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(b) Find the equation of the tangent to circle Q at the point  $(-4, 1)$ .

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(c) The tangent in (b) intersects circle P in two points. Find the  $x$ -coordinates of the points of intersection, expressing your answers in the form  $a \pm b\sqrt{3}$ .

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[END OF QUESTION PAPER]