X056/303

NATIONAL QUALIFICATIONS 2001

THURSDAY, 17 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

ALL questions should be attempted.

Marks

1. (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

   (*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

2. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}, \quad x > 0$.

   Find the equation of the tangent at the point where $x = 4$. **6**

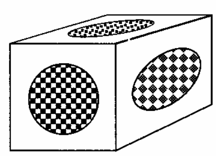
3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1·5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

   (*a*) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

   Let u_n and u_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving u_{n+1} and u_n. **2**

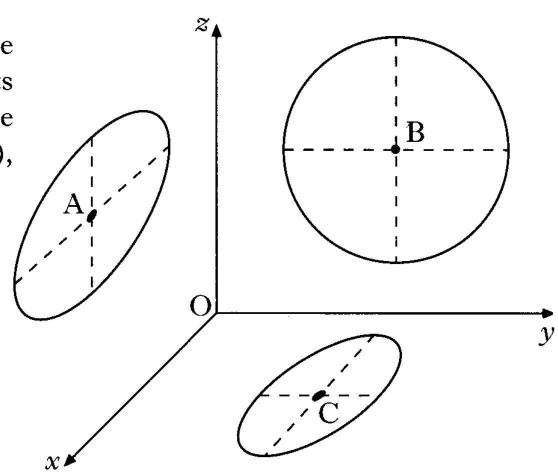
   (*b*) Find the date and the amount of the final payment. **4**

4. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.



   The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0).

   Find the size of angle ABC. **7**



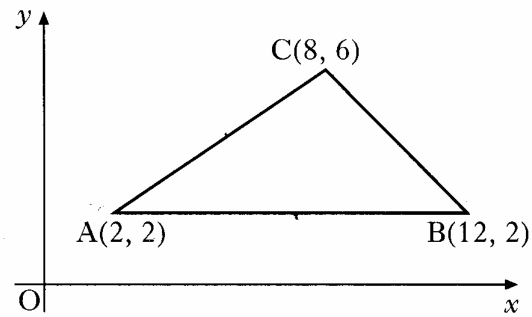
[Turn over

Marks

5. Express $8\cos x° - 6\sin x°$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 < a < 360$. **4**

6. Find $\int \frac{(x^2-2)(x^2+2)}{x^2}dx,\ \ x \neq 0$ **4**

7. Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).



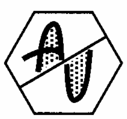
(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

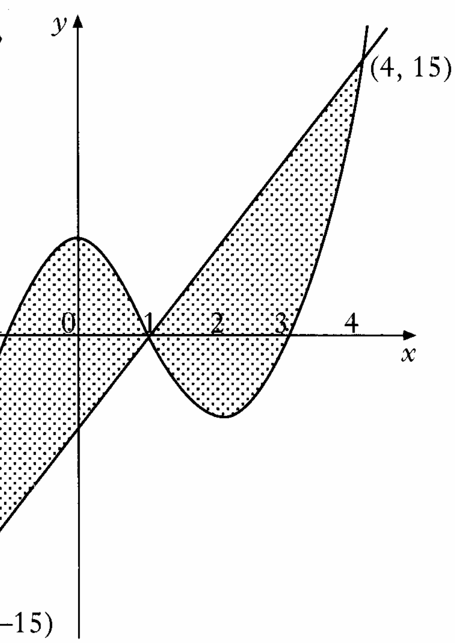
(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

(*c*) Find the point of intersection of lines l_1 and l_2. **1**

(*d*) Hence find the equation of the circle passing through A, B and C. **2**

8. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.





A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point (1, 0) is the centre of half-turn symmetry.

Calculate the total shaded area. **7**

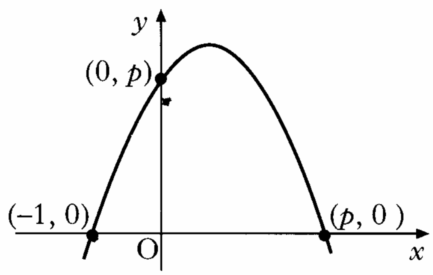
9. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and half hours for the area of the forest fire to double, find the value of the constant k. **3**

10. A curve for which $\dfrac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$.

Find y in terms of x. **4**

11. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.



(*a*) Show that the equation of the parabola is $y = p + (p-1)x - x^2$. **3**

(*b*) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**

[*END OF QUESTION PAPER*]

