

2002 Mathematics

Higher – Paper 1

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.
4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✘).
5.
 - The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
 - Only the mark should be written, **not** a fraction of the possible marks.
 - These marks should correspond to those on the question paper and these instructions.
6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.
Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.
7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

cont/

8. Do not penalise:
 - working subsequent to a correct answer
 - omission of units
 - bad form
 - legitimate variations in numerical answers
 - correct working in the “wrong” part of a question
9. No marks should be awarded for a part of an answer which shows a complete misunderstanding of any fundamental principle or complete ignorance of any process involved in that part.
10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.
11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a PA referral. Write PA at the top left of the front cover of the script and complete the PA referral sheet. This reference must be restricted to genuine cases of difficulty. **Also**, write the letters “PA” (in red) on Form Ex6 immediately after the candidate’s name.
12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.
13. Do not write any comments on the scripts. A summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

- 1 **Tick** correct working.
- 2 Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
- 3 **Do not** write marks as fractions.
- 4 Put each mark **at the end** of the candidate’s response to the question.
- 5 **Follow through** errors to see if candidates can score marks subsequent to the error.
- 6 **Do not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓	The tick. You are not expected to tick every line but of course you must check through the whole of a response.	Marks being allotted e.g. (•) would not normally be shown on scripts		
✗	The cross and underline. Underline an error and place a cross at the end of the line.	$\frac{dy}{dx} = 4x - 7$ ✓ • $4x - 7 = 0$ ✗ $x = \frac{7}{4}$ ✗ $y = 3\frac{7}{8}$ ✗ •		2
✓✗	The tick-cross. Use this to show correct work where you are following through subsequent to an error.	$C = (1, -1)$ ✗ $m = \frac{3 - (-1)}{4 - 1}$ $m_{rad} = \frac{4}{3}$ ✗ • $m_{tgt} = \frac{-1}{\frac{4}{3}}$ $m_{tgt} = -\frac{3}{4}$ ✗ • $y - 3 = -\frac{3}{4}(x - 2)$ ✗ •		3
∧	The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.	$x^2 - 3x = 28$ ✓ • $x = 7$ ∧ ✗		1
~	The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).			
RE	Repeated error (which would generally not be penalised within the same question).	$\sin 2A = 2 \sin A \cos A$ ✓ • $= 2 \times \frac{1}{3} \times \frac{3}{4}$ ✗ $= \frac{1}{2}$ ✗ • $\cos 2A = \cos^2 A - \sin^2 A$ ✓ • $= \frac{9}{16} - \frac{1}{9}$ RE ✗ • $= \frac{65}{144}$ ✗ •		5
EA	Eased. Where working is found correct whilst following through subsequent to an error, the working has been eased sufficiently for a mark not to be awarded.	$\log_3(x - 2) = 1$ ✗ $(x - 2) = 3^1$ ✗ • $x - 2 = 3$ $x = 5$ EA ✗		1

All of these are to help us be more consistent and accurate.

It goes without saying that however accurate you are in marking, it is to no avail unless you have added the marks up correctly. Please double check totals!!

	Give 1 mark for each •	Illustrations for awarding each •															
1	The point P(2, 3) lies on the circle $(x+1)^2 + (y-1)^2 = 13$. Find the equation of the tangent to the circle at P.	4															
1	<p>2.4.1, 1.1.10 CN. C 02/35</p> <p>ans: $2y+3x=12$ 4 marks</p> <p>•¹ ic : interpret centre from equ of circle •² ss : know to find gradient of radius •³ ss : know find perpendicular gradient •⁴ ic : state equation of tangent</p>	<p>•¹ $C = (-1, 1)$ ic</p> <p>•² $m_{rad} = \frac{2}{3}$ ss</p> <p>•³ $m_{igt} = -\frac{3}{2}$ ss</p> <p>•⁴ $y - 3 = -\frac{3}{2}(x - 2)$ ic</p>															
<p style="text-align: right;">Notes</p> <p>1 •⁴ is not available unless an attempt has been made to find a perpendicular gradient.</p> <p>2 •², •³ and •⁴ are not available to candidates who incorrectly attempt to use calculus.</p> <p>3 Please make a PA referral for any candidate who uses implicit differentiation correctly.</p> <p>4 •⁴ is not available unless the gradient e.g. $\left(-\frac{1}{3}\right)$ has been simplified.</p> <p>Example</p> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: fit-content; margin-left: 20px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px;">$C = (1, -1)$</td> <td style="padding: 2px; text-align: center;">✗</td> <td></td> </tr> <tr> <td style="padding: 2px;">$m_{rad} = 4$</td> <td style="padding: 2px; text-align: center;">✓</td> <td style="padding: 2px;">•2</td> </tr> <tr> <td style="padding: 2px;">$m_{igt} = -\frac{1}{4}$</td> <td style="padding: 2px; text-align: center;">✓</td> <td style="padding: 2px;">•3</td> </tr> <tr> <td style="padding: 2px;">$y - 3 = -\frac{1}{4}(x - 2)$</td> <td style="padding: 2px; text-align: center;">✓</td> <td style="padding: 2px;">•4</td> </tr> <tr> <td colspan="3" style="padding: 2px; text-align: right;">3 marks given</td> </tr> </table> </div>			$C = (1, -1)$	✗		$m_{rad} = 4$	✓	•2	$m_{igt} = -\frac{1}{4}$	✓	•3	$y - 3 = -\frac{1}{4}(x - 2)$	✓	•4	3 marks given		
$C = (1, -1)$	✗																
$m_{rad} = 4$	✓	•2															
$m_{igt} = -\frac{1}{4}$	✓	•3															
$y - 3 = -\frac{1}{4}(x - 2)$	✓	•4															
3 marks given																	
5																	

Give 1 mark for each •		Illustrations for awarding each •	
2	The point Q divides the line joining P(-1, -1, 0) to R(5, 2, -3) in the ratio 2 : 1. Find the coordinates of Q.	3	
2	<p>3.1.6 NC C 02/64</p> <p>ans: (3, 1, -2) 3 marks</p> <p>•¹ pd : find vector components •² ss : use parallel vectors •³ pd : process vectors</p>	<p>•¹ $\vec{PR} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$ pd</p> <p>•² $\vec{PQ} = \frac{2}{3}\vec{PR}$ ss</p> <p>•³ $Q = (3, 1, -2)$ pd</p>	
<p>Note</p> <p>1 An incorrect ratio loses •²</p> <p>2 Treat $Q = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ as bad form.</p> <p>3 The coordinates of Q are only worth •³. So an answer with no working is only worth 1 mark (ie •³).</p> <p>4 The justification (ie •¹ and •²) may take the form of a diagram eg below are two examples of a diagram and the answer for Q. Each may be awarded 3 marks.</p>			
<p>Further examples</p>			
1	2	3	
<p>•¹ $\vec{PR} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$ ✓ •¹</p> <p>•² $\vec{PQ} = \frac{1}{3}\vec{PR}$ ✗</p> <p>•³ $Q = (1, 0, -1)$ ✗ •³ 2 marks</p>	<p>•¹ $q = \frac{1p+2r}{3}$ ✓ •¹</p> <p>•² substitute col v. ✓ •²</p> <p>•³ $Q = (3, 1, -2)$ ✓ •³ 3 marks</p>	<p>•¹ $q - p = 2(r - q)$ ✓ •¹</p> <p>$\left[\text{maybe preceded by } \vec{PQ} = 2\vec{QR} \right]$ ✓ •²</p> <p>•² substitute col v. ✓ •²</p> <p>•³ $Q = (3, 1, -2)$ ✓ •³ 3 marks</p>	
6			

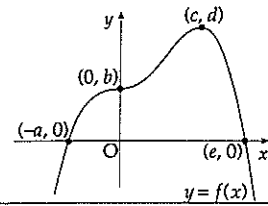
	Give 1 mark for each •	Illustrations for awarding each •
3	<p>Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.</p> <p>(a) Find expressions for</p> <p>(i) $f(g(x))$</p> <p>(ii) $g(f(x))$.</p>	<p>2</p> <p>5</p>
3	<p>1.2.6, 2.3.3+ CN C 02/5</p> <p>a ans: $\sin(2x^\circ)$ & $2\sin(x^\circ)$ 2 marks</p> <p>b $0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$ 5 marks</p> <p>•¹ ic : interpret $f(g(x))$</p> <p>•² ic : interpret $g(f(x))$</p> <p>•³ ss : equate for intersection</p> <p>•⁴ ss : substitute for $\sin 2x$</p> <p>•⁵ pd : extract a common factor</p> <p>•⁶ pd : solve a 'common factor' equation</p> <p>•⁷ pd : solve a 'linear' equation</p>	<p>•¹ $\sin(2x^\circ)$ ic</p> <p>•² $2\sin(x^\circ)$ ic</p> <p>•³ $2\sin(2x^\circ) = 2\sin(x^\circ)$ ss</p> <p>•⁴ appearance of $2\sin(x^\circ)\cos(x^\circ)$ ss</p> <p>•⁵ $2\sin(x^\circ)(2\cos(x^\circ) - 1)$ pd</p> <p>•⁶ $\sin(x^\circ) = 0$ and $0, 180, 360$ pd</p> <p>•⁷ $\cos(x^\circ) = \frac{1}{2}$ and $60, 300$ pd</p> <p>or</p> <p>•⁶ $\sin(x^\circ) = 0$ and $\cos(x^\circ) = \frac{1}{2}$ pd</p> <p>•⁷ $0, 60, 180, 300, 360$ pd</p>
	<p>Examples</p> <p style="text-align: center;">1</p> <div style="border: 1px solid black; border-radius: 15px; padding: 10px;"> <p>$2\sin(2x^\circ) = 2\sin(x^\circ)$ ✓ •3</p> <p>$2 \times 2\sin x^\circ \cos x^\circ = 2\sin x^\circ$ ✓ •4</p> <p>$2\cos(x^\circ) = 1$ ✗</p> <p>$\cos x^\circ = \frac{1}{2}$ ✗</p> <p>$x = 60, 300$ ✓ •7</p> <p style="text-align: right;">3 marks</p> </div> <p style="text-align: center;">2</p> <div style="border: 1px solid black; border-radius: 15px; padding: 10px;"> <p>$g(f(x)) = \sin(2x^\circ)$ ✗</p> <p>$f(g(x)) = 2\sin(x^\circ)$ ✓ •2</p> <p>$2 \times 2\sin(x^\circ) = \sin(2x^\circ)$ ✓ •3</p> <p>appearance of $2\sin(x^\circ)\cos(x^\circ)$ ✓ •4</p> <p>$2\sin(x^\circ)(2 - \cos(x^\circ)) = 0$ ✓ •5</p> <p>$\sin(x^\circ) = 0$ and $0, 180, 360$ ✓ •6</p> <p>$\cos(x^\circ) = 2$ and no solution ✗ eased</p> <p style="text-align: right;">5 marks</p> </div> <p>Examples : some endings</p> <p style="text-align: center;">3</p> <div style="border: 1px solid black; border-radius: 15px; padding: 10px;"> <p>.....</p> <p>$2\sin x^\circ \cos x^\circ = 0$</p> <p>$\sin 2x^\circ = 0$</p> <p>$2x = 0, 180, 360, \dots, 720$</p> <p>$x = 0, 90, 180, 270, 360$ ✓ •7</p> </div> <p style="text-align: center;">4</p> <div style="border: 1px solid black; border-radius: 15px; padding: 10px;"> <p>.....</p> <p>$2\sin x^\circ \cos x^\circ = 0$</p> <p>$\sin x^\circ = 0$ and $\cos x^\circ = 0$</p> <p>$x = 0, 180, 360$ and $90, 270$ ✓ •7</p> </div>	<p>Notes</p> <p>1 1 mark may be given for $f(2x)$ and $g(\sin x^\circ)$ where final f and g are both wrong.</p> <p>2 Example 2 illustrates an <i>easing</i> where a mark is not awarded (although the working is correct). Similarly, solving $\cos(x^\circ) = 1 \dots x = 0$ or 360 would be considered as easing the working.</p>

Give 1 mark for each •		Illustrations for awarding each •	
4	Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x -axis.	4	
4	<p>1.3.10, 1.1.3 NC C 02/15</p> <p>ans: (2, 4) 4 marks</p> <p>•¹ sp : know to diff and differentiate •² pd : process gradient from angle •³ ss : equate equivalent expressions •⁴ pd : solve and complete</p>	<p>•¹ $\frac{dy}{dx} = 4x - 7$ sp</p> <p>•² $m_{\text{tang}} = \tan 45^\circ = 1$ pd</p> <p>•³ $4x - 7 = 1$ ss</p> <p>•⁴ (2,4) pd</p>	
<p>Notes</p> <p>1 Simply stating that $\tan 45^\circ = 1$ earns no marks.</p> <p>Examples</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: 45%;"> <p style="text-align: center;">1</p> <p>$\frac{dy}{dx} = 4x - 7$ ✓ •1 $4x - 7 = 0$ ✗ $x = \frac{7}{4}$ $y = 3\frac{7}{8}$ ✗ •4 2 marks</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: 45%;"> <p style="text-align: center;">2</p> <p>$y = x + c$ ✓ •1 $2x^2 - 8x + 10 - c = 0$ ✓ •2 $\Delta = 64 - 8(10 - c) = 0$ ✓ •3 $c = 2$ and point = (2,4) ✓ •4 4 marks</p> </div> </div>			
5	In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$.	4	
5	<p>2.3.2 NC C 02/32</p> <p>ans: proof 4 marks</p> <p>•¹ pd : process the missing sides •² ss : expand •³ pd : substitute •⁴ pc : process & complete proof</p>	<p>•¹ $AC = \sqrt{2}$ and $BC = \sqrt{10}$ pd stated or implied by •³</p> <p>•² $\sin(a+b) = \sin a \cos b + \cos a \sin b$ ss</p> <p>•³ $\frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$ pd</p> <p>•⁴ $\frac{4}{\sqrt{20}} = \dots = \frac{2}{\sqrt{5}}$ pc</p>	
<p>Notes</p> <p>1 Partial credit: 2 marks may be given for either of $\frac{1}{\sqrt{2}} \cos b + \frac{1}{\sqrt{2}} \sin b$ or $\frac{3}{\sqrt{10}} \sin a + \frac{1}{\sqrt{10}} \cos a$.</p> <p>2 At •⁴ the completion of the proof might be as $\frac{4}{\sqrt{20}} = \frac{4}{\sqrt{4 \cdot 5}} = \frac{2}{\sqrt{5}}$ OR $\frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$.</p>			

Give 1 mark for each •

Illustrations for awarding each •

6 The graph of a function f intersects the x -axis at $(-a, 0)$ and $(e, 0)$ as shown. There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d) . Sketch the graph of the derived function f' .

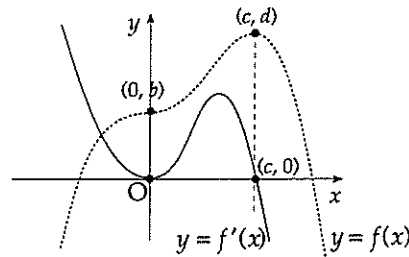


3

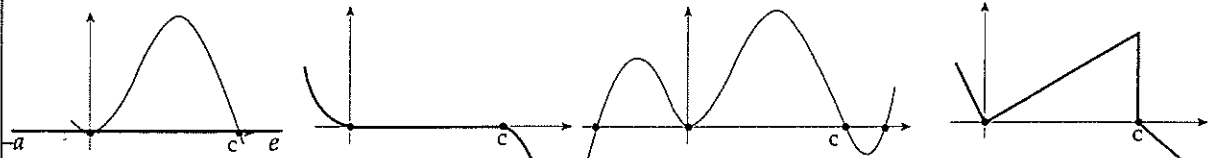
6 1.2.4 CN C 02/8
ans: sketch 3 marks

- ¹ ic : interpret stationary points
- ² ic : interpret main body of f
- ³ ic : interpret tails of f

- ¹ roots at 0 and c
(accept a statement to this effect) ic
- ² min at LH root, max between roots ic
- ³ both 'tails' correct ic



Examples

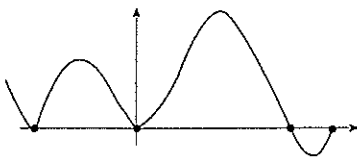


- ✓ •1
- ✓ •2
- X •3

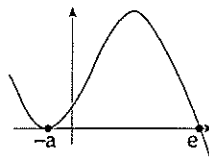
- ✓ •1
- X •2
- ✓ •3

- ✓ •1
- ✓ •2
- X •3

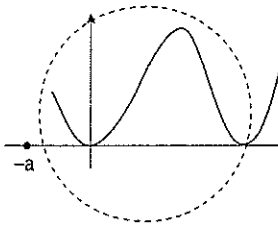
- ✓ •1
- X •2
- ✓ •3



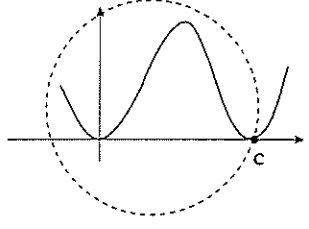
award 1 mark



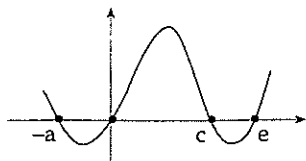
award 1 mark



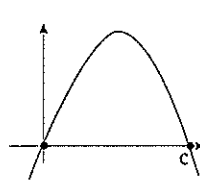
award 1 mark



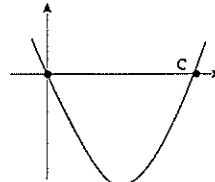
award 2 marks



award 1 mark

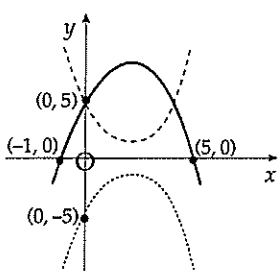
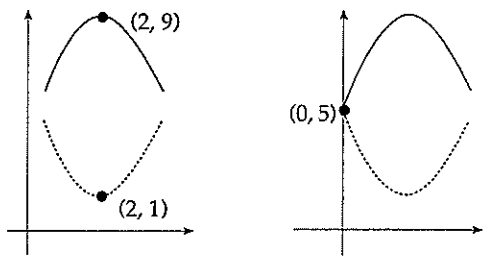


- ✓ •1
- X •2
- X •3



- ✓ •1
- X •2
- X •3

9

	Give 1 mark for each •	Illustrations for awarding each •
7	(a) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. (b) On the same diagram sketch (i) the graph of $y = f(x)$ (ii) the graph of $y = 10 - f(x)$. (c) Find the range of values of x for which $10 - f(x)$ is positive.	2 4 1
7	1.2.8 +, 1.2.4 a ans: $a = 2, b = 1$ 2 marks b ans: sketch 4 marks c ans: sketch 1 mark • ¹ pd : process eg completing the square • ² pd : process eg completing the square • ³ ic : interpret minimum • ⁴ ic : interpret y-intercept • ⁵ ss : reflect in x -axis • ⁶ ss : translate parallel to y -axis • ⁷ ic : interpret graph	<ul style="list-style-type: none"> •¹ $a = 2$ pd •² $b = 1$ pd •³ any 2 from parabola; min tp(2,1); (0,5) ic •⁴ the remaining one from above list ic •⁵ reflecting in x-axis ss •⁶ translating +10units \parallel^{el} to y-axis ss •⁷ $(-1,5)$ ie $-1 < x < 5$ ic <p>Answer</p>  <p>Note</p> <ol style="list-style-type: none"> 1 At •⁷, do not penalise $-1 \leq x \leq 5$ 2 If transformations done in wrong order, then •⁶ is not available. 3 Accept phrase "reflect in x-axis and then translate up 10 units" for 1 mark (in lieu of •⁵ and •⁶). 4 Either of the two diagrams below illustrates the minimum requirements for •⁵ and •⁶.  <ol style="list-style-type: none"> 5 •⁵ and •⁶ are still available for candidates who do not produce a parabola at •³/⁴ stage. However a straight line would be a case of easing the working and so for a straight line a maximum of 1 mark (from •⁵ & •⁶) could be awarded for a correct reflection and translation.

	Give 1 mark for each •	Illustrations for awarding each •	
8	The diagram shows the graph of a cosine function from 0 to π . (a) State the equation of the graph. (b) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B. Find the coordinates of B.		1 3

8 1.2.3, 2.3.1 NC C 02/14

a ans: $y = 2\cos 2x$ 1 mark

b ans: $B\left(\frac{7\pi}{12}, -\sqrt{3}\right)$ 3 marks

- ¹ ic : interpret graph
- ² ss : equate equal parts
- ³ pd : solve linear trig equation in radians
- ⁴ ic : interpret result

- ¹ $2 \cos 2x$ ic
- ² $2 \cos 2x = -\sqrt{3}$ ss
- ³ $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$ pd
- ⁴ $x = \frac{7\pi}{12}$ ic

Notes

- 1 •² is available for whatever function is obtained in (a).
- 2 As a consequence of an incorrect function at •¹, for •⁴ to be available, the two roots from •³ must lie as follows:
 1 root lies in the interval $0 \dots \pi/2$ and
 1 root lies in the interval $\pi/2 \dots \pi$.
- 3 $x = 105^\circ$ does not earn •⁴

Examples

1

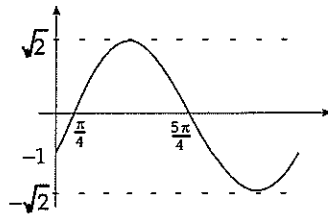
$y = 2 \cos x$ ✗
 $2 \cos x = -\sqrt{3}$ ✓ •2
 $\cos x = -\frac{\sqrt{3}}{2}$
 $x = \frac{5\pi}{6}, \frac{7\pi}{6}$ ✓ •3
 $x = \dots$ ✗
 2 marks max. (The answers bear no relation to diagram)

2

$y = 2 \cos x$ ✗
 $2 \cos x = -\sqrt{3}$ ✓ •2
 $\cos x = -\frac{\sqrt{3}}{2}$
 $x = 150^\circ, 210^\circ$ ✓ •3
 $x = \dots$ ✗
 2 marks max. (The answers bear no relation to diagram)

	Give 1 mark for each •	Illustrations for awarding each •
9	(a) Write $\sin(x) - \cos(x)$ in the form $k \sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$.	4
	(b) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x -axis and the y -axis.	3

9	3.4.1	NC C	02/81	
a	ans: $\sqrt{2} \sin(x - \frac{\pi}{4})$	4 marks		<ul style="list-style-type: none"> •¹ $k \sin(x) \cos(a) - k \cos(x) \sin(a)$ explicitly stated •² $k \cos(a) = 1$ and $k \sin(a) = 1$ explicitly stated
b	ans: sketch	3 marks		<ul style="list-style-type: none"> •³ $k = \sqrt{2}$ pd •⁴ $a = \frac{\pi}{4}$ pd •⁵ correct shape of graph (<i>ie</i> \sin) but not passing through the origin ic •⁶ graph lies between $\sqrt{2}$ and $-\sqrt{2}$ ic •⁷ $(\frac{\pi}{4}, 0), (\frac{5\pi}{4}, 0), (0, -1)$ pd
	<ul style="list-style-type: none"> •¹ ss : know to expand and expand •² ic : compare coefficients •³ pd : write down the value of k •⁴ pd : process a •⁵ ic : sketch a sine curve •⁶ ic : int/com max. and min. values •⁷ pd : process intercepts 			

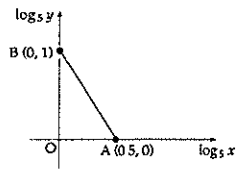


Notes

- 1 No justification is required for $k (= \sqrt{2})$
- 2 Do not penalise degrees at the •⁴ stage
- 3 •⁷ is only available for answers in radians.
- 4 If k is worked out to be 1, •⁶ is not available (eased)
- 5 Do not penalise graphs which go beyond the interval $0 \dots 2\pi$.

Example

$t^2 = 1^2 + 1^2 \Rightarrow t = \sqrt{2}$	✓	•1
$\sin x - \cos x = \sqrt{2}(\sin x \times \frac{1}{\sqrt{2}} - \cos x \times \frac{1}{\sqrt{2}})$	✓	•2
$= \sqrt{2}(\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4})$	✓	•3
$= \sqrt{2} \sin(x - \frac{\pi}{4})$	✓	•4

Give 1 mark for each •		Illustrations for awarding each •	
10	(a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$.		2
	(b) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} dx$.		1
10	3.2.2 CN AB 02/62		
a	ans: $-\frac{3}{2}x^2(8 - x^3)^{-\frac{1}{2}}$ 2 marks	• ¹ $\frac{1}{2}(8 - x^3)^{-\frac{1}{2}}$	pd
b	ans: $-\frac{2}{3}(8 - x^3)^{\frac{1}{2}} + c$ 1 mark	• ² $\dots \times -3x^2$	pd
	• ¹ pd : process differentiation • ² pd : use the chain rule • ³ ic : interpret answer from (a)	• ³ $-\frac{2}{3}f(x)$ or $-\frac{2}{3}(8 - x^3)^{\frac{1}{2}}$	ic
11	The graph illustrates the law $y = kx^n$. If the straight line passes through A(0.5, 0) and B(0, 1), find the values of k and n .		4
11	3.3.7 NC AB 02/77		
	ans : $y = 5x^{-2}$ 4 marks	• ¹ $\log_5 y = -2(\log_5 x) + 1$	ic
	• ¹ ic : interpret graph • ² ss : use log laws • ³ ss : use log laws • ³ pd : solve log equation	• ² $\log_5 y = \log_5 x^{-2} + \dots$	ss
		• ³ $\dots + \log_5 5$	pd
		• ⁴ $y = 5x^{-2}$	pd
Note Do not accept $\frac{1}{-\frac{1}{2}}$ for the value of n .			
Examples			
1	No 'theory', just gradient = -2 so $n = -2$ ✓ •3 intercept = 1 $\log_5 k = 1$ so $k = 5$ ✓ •4 2 marks max.	2	$\log_5 y = -2(\log_5 x) + 1$ ✓ •1 $\log_5 y + \log_5 x^2 = 1$ ✓ •2 $\log_5(yx^2) = 1$ ✓ •3 $yx^2 = 5$ so $k = 5, n = -2$ ✓ •4 4 marks
3	$\log y = (\log kx^n)$ ✓ •1 $\log y = n \log x + \log k$ ✓ •2 gradient = -2 so $n = -2$ ✓ •3 intercept = 1 $\log_5 k = 1$ so $k = 5$ ✓ •4 4 marks	4	$\log y = (\log kx^n)$ ✓ •1 $\log y = n \log x + \log k$ ✓ •2 followed by sim. equations $1 = n \times 0 + \log k$ $0 = n \times (\frac{1}{2}) + \log k$ leads to $n = -2$ ✓ •3 $\log_5 k = 1$ so $k = 5$ ✓ •4 4 marks

Give 1 mark for each •

Illustrations for awarding each •

for the Mathematics with Statistics paper

Replacing Maths qu 2, 9 & 10,11 .

- 2 In a survey for a supermarket, 1000 customers were asked to answer these two questions:
 A : Would you use the supermarket more often if there were facilities for selling petrol?
 B : Would you use the supermarket more often if there were facilities for processing films?
 Some of the results are recorded in the table below.

		question B		
		yes	no	total
question A	yes	102		510
	no	84		
	total			1000

For this set of customers calculate

- (i) $P(\text{customer responded 'yes' to question A})$.
- (ii) $P(\text{customer responded 'no' to both questions})$.
- (iii) $P(\text{customer responded 'no' to at least one question})$.

4

- 9 A Probus Club went in three buses to a Garden Festival. The frequency distribution of the ages of those on the trip was as follows:

<i>age</i>	60	61	62	63	64	65	66	67	68	69	70	75	77
<i>frequency</i>	10	12	11	8	16	14	9	13	18	15	6	1	1

Would either of the two eldest people be considered as an outlier for this distribution?

5

- 11 Andrew throws a dart at a circular target of radius a cm. The random variable X represents the distance a dart lands from the centre of the circle. All points in the circle are equally likely to be hit by the dart.

- (a) Find $P(X < x)$, the probability that a dart lands less than x cm from the centre of the circle.
- (b) Show that the probability density function for the distribution of the distance of a dart from the centre is

$$f(x) = \begin{cases} \frac{2x}{a^2} & 0 \leq x \leq a \\ 0 & \text{otherwise.} \end{cases}$$

- (c) Find the mean and variance of this distribution.

2

1

5

	Give 1 mark for each •	Illustrations for awarding each •
2	<p>4.2.3 CN C 02/S7</p> <p>ans : 0.51, 0.406, 0.898 4 marks</p> <ul style="list-style-type: none"> •¹ ic : complete table •² ic : interpret table •³ ic : interpret the table •⁴ ss : know to add relevant numbers 	<ul style="list-style-type: none"> •¹ complete relevant bits of table <i>i.e.</i> (490, 406) <i>s/i</i> by •³ ic •² 0.51 ic •³ 0.406 <i>pd</i> •⁴ 0.898 ss
9	<p>4.1.4 NC AB 02/S2</p> <p>ans : only eldest 5 marks</p> <ul style="list-style-type: none"> •¹ ss : know how to find Qs eg c.f.s •² <i>pd</i> : process median •³ <i>pd</i> : process quartiles •⁴ <i>pd</i> : process fences •⁵ <i>ic</i> : interpret value of fence 	<ul style="list-style-type: none"> •¹ cum.frequency column <i>s/i</i> by •² or •³ ss •² $Q_1 = 63$ <i>pd</i> •³ $Q_3 = 68$ <i>pd</i> •⁴ fence = $75\frac{1}{2}$ <i>pd</i> •⁵ comment: only the eldest is outlier <i>ic</i>
11	<p>4.2.3, 4.3.3,4 CN AB 02/S14</p> <p>a ans : $\frac{x^2}{a^2}$ 2 marks</p> <p>b ans : $\frac{2x}{a^2}$ 1 mark</p> <p>c ans : $\frac{2}{3}a, \frac{1}{18}a^2$ 5 marks</p> <ul style="list-style-type: none"> •¹ ss : know how to find probability •² <i>pd</i> : process probability •³ ss : know that $f(x) = F'(x)$ •⁴ ss : know how to find mean •⁵ <i>pd</i> : process $E(X)$ •⁶ ss : know how to find variance •⁷ <i>pd</i> : process $E(X^2)$ •⁸ <i>pd</i> : process variance 	<ul style="list-style-type: none"> •¹ probability = $\frac{\text{favourable area}}{\text{possible area}}$ <i>s/i</i> ss •² probability = $\frac{x^2}{a^2}$ <i>pd</i> •³ $f(x) = F'(x) = \frac{2x}{a^2}$ ss •⁴ mean = $E(X) = \int_0^a \frac{2x^2}{a^2} dx$ ss •⁵ $\frac{2}{3}a$ <i>pd</i> •⁶ $VarX = E(X^2) - (E(X))^2$ ss •⁷ $E(X^2) = \int_0^a \frac{2x^3}{a^2} dx = \frac{1}{2}a^2$ <i>pd</i> •⁸ $\frac{1}{18}a^2$ <i>pd</i>