

X100/301

NATIONAL
QUALIFICATIONS
2002

MONDAY, 27 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

| $f(x)$ | $f'(x)$ |
|-----------|--------------|
| $\sin ax$ | $a \cos ax$ |
| $\cos ax$ | $-a \sin ax$ |

Table of standard integrals:

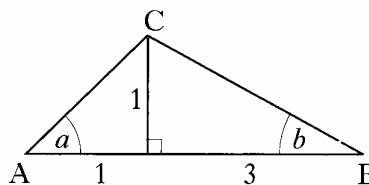
| $f(x)$ | $\int f(x) dx$ |
|-----------|----------------------------|
| $\sin ax$ | $-\frac{1}{a} \cos ax + C$ |
| $\cos ax$ | $\frac{1}{a} \sin ax + C$ |

ALL questions should be attempted.

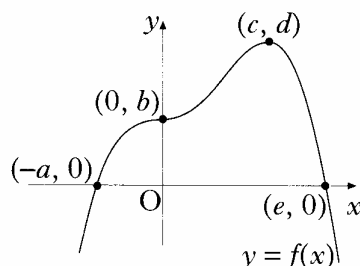
Marks

1. The point $P(2, 3)$ lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P . 4
2. The point Q divides the line joining $P(-1, -1, 0)$ to $R(5, 2, -3)$ in the ratio $2 : 1$. Find the coordinates of Q . 3
3. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.
- (a) Find expressions for:
- (i) $f(g(x))$;
- (ii) $g(f(x))$. 2
- (b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. 5
4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x -axis. 4

5. In triangle ABC , show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$. 4



6. The graph of a function f intersects the x -axis at $(-a, 0)$ and $(e, 0)$ as shown. There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d) . Sketch the graph of the derived function f' . 3

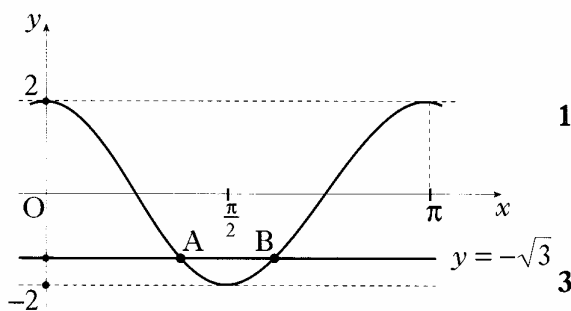


[Turn over for Questions 7 to 11 on Page four

7. (a) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. 2
 (b) On the same diagram sketch:
 (i) the graph of $y = f(x)$;
 (ii) the graph of $y = 10 - f(x)$. 4
 (c) Find the range of values of x for which $10 - f(x)$ is positive. 1

8. The diagram shows the graph of a cosine function from 0 to π .

- (a) State the equation of the graph. 1
 (b) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B. 3
 Find the coordinates of B.

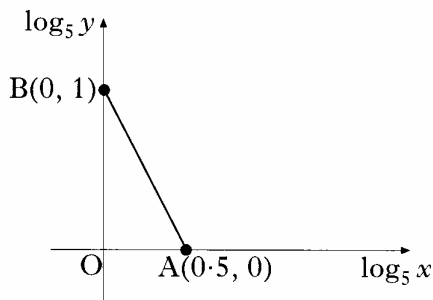


9. (a) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. 4
 (b) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x -axis and the y -axis. 3

10. (a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. 2

- (b) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} dx$. 1

11. The graph illustrates the law $y = kx^n$.
 If the straight line passes through $A(0.5, 0)$ and $B(0, 1)$, find the values of k and n . 4



[END OF QUESTION PAPER]