SCOTTISH QUALIFICATIONS AUTHORITY

2003 Mathematics

Higher

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✔).This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.
   Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will be indicated in the marking instructions.

cont/

8. Do not penalise:
   - working subsequent to a correct answer
   - omission of units
   - bad form
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question

9. No piece of work should be scored through - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 **Do not write any comments on the scripts**. A summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3 Do **not** write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

	Give 1 mark for each •	Illustrations for awarding each •
1	Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x+y-1=0$. 3	
1	**1.1.9, 1.1.7 CN C 03/1** **ans:** $x-4y+13=0$ **3 marks** •1 ic : interpret gradient from linear equ. •2 ic : find perp. gradient •3 ic : state equation of line	•1 $m=-4$ **stated or implied by •2** •2 $m_{perp}=\frac{1}{4}$ •3 $y-3=\frac{1}{4}(x-(-1))$

Notes
1 •3 is only available following an attempt to find the perpendicular gradient.
2 Wrong answer with no working gains no marks.

Example 1

$m=4$ •1 ✗
$m_{perp}=-\frac{1}{4}$ •2 ✓ f.t.
$y-3=-\frac{1}{4}(x--1)$ •3 ✓ f.t.
2 marks given

Example 2

$m=-4$ •1 ✓
•2 ✗
$y-3=-4(x--1)$ •3 ✗ no perp. grad.
1 mark given

Example 3

$y=-4x-1$ ignore the error (of –1)
$m=-4$ •1 ✓
etc

Example 4

$y=-4x+1$ •1 ✗
$m=4$ •2 ✗
$y-3=4(x--1)$ •3 ✓ f.t. BOD
1 mark awarded

Example 5

$m=4$
$y-3=4(x--1)$ 0 marks given

Example 6

$y=-4x+1$
$\frac{dy}{dx}=-4$ is acceptable for •1
$m=-4$

Example 7

$y-3=\frac{1}{4}(x--1)$ may be awarded 2 marks,
or 1 mark being lost through
$x-4y=-13$ lack of communication

Example 8

$m=-4$ •1 ✓
$m=\frac{1}{4}$ •2 ✓
$y=\frac{1}{4}x+c$
$3=\frac{1}{4}\times(-1)+c$
$c=3+\frac{1}{4}$ •3 ✓
3 marks given

Example 9

$m=\frac{1}{4}$
and nothing else 0 marks given

Example 10

$m=-\frac{1}{4}$ •1 ✗
$y-3=-\frac{1}{4}(x--1)$ •2 ✗
•3 ✓ b.o.d for perp gr.
1 mark awarded

	Give 1 mark for each •	Illustrations for awarding each •

2 (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x+a)^2 + b$. **2**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

2 **1.2.8 + CN C 03/new**

(a) ans: $(x+3)^2 + 2$ **2 marks**

(b) ans: sketch **2 marks**

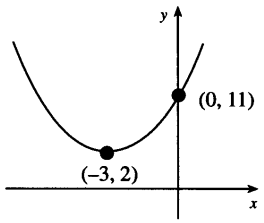
•1 ic : start to complete square
•2 pd : finish completing the square

•3 ic : sketch
•4 ic : sketch

•1 $(x+3)^2$
•2 +2

•3 *any two from*
* *U – parabola*
* *minimum at* (–3,2)
* ***intercept on y – axis at*** (0,11)

•4 ***all three of the above facts***



Notes
1 2 marks may be awarded for stating a = 3 and b = 2
2 accept the information about (–3, 2) and (0, 11) in the body of the working provided the sketch shows a parabola in a consistent position.

(a)
$(x+3)^2 + 2$ •1 ✓ •2 ✓
(b)
For a U parabola through (0, 11) and (3, 2) •3 ✓ •4 ✗
3 mark given

(a)
$(x-3)^2 + 2$ •1 ✗ •2 ✓
(b)
For a U parabola through (0, 11) and (3, 2) •3 ✓ •4 ✓
3 marks given

(a)
$(x+a)^2 + b = x^2 + 2ax + a^2 + b$
$2a = 6$ .....$a = 3$ •1 ✓
$a^2 + b = 11$.....$b = 2$ •2 ✓

(a)
$(x+3)^2 + 20$ •1 ✓ •2 ✗
(b)
For a U parabola through (0, 11) and the minimum in the right position but marked as (-3, 20) •3 ✓ •4 ✗
2 marks given

	Give 1 mark for each •	Illustrations for awarding each •
3	Vectors u and v are defined by $u=3i+2j$ and $v=2i-3j+4k$. Determine whether or not u and v are perpendicular to each other. **2**	
3	**3.1.1/ .9/ .10 CN C 03/53** **ans : vectors are perpendicular** **2 marks** •1 ss : use scalar product and get zero •2 pd : process	•1 for perpendicularity $u.v=0$ •2 $\begin{pmatrix}3\\2\\0\end{pmatrix}.\begin{pmatrix}2\\-3\\4\end{pmatrix}=6-6+0=0$

Example 1

•1 $\begin{pmatrix}3\\2\\0\end{pmatrix}.\begin{pmatrix}2\\3\\4\end{pmatrix}$ •1 ✗

•2 $6+6+0=12$
so u and v not perp. •2 √ f.t.

1 mark given

Example 2

$\cos(\theta)=a_1b_1+a_2b_2+a_3b_3$ •1 ✗
$=6-6+0$
$=0$
$\theta=90$ •2 √ f.t.

1 mark given

Example 3

$\cos(\theta)=\dfrac{6-6+0}{|u||v|}$ •2 ✓
$=0$
$\theta=90$ •1 ✓

2 marks given

Example 4

$(3i+2j).(2i-3j+4k)$
$=6i.i-9i.j+12i.k+4j.i-6j.j+8j.k$ •2 ✓
$=6-6$
$=0$
so u,v perpendicular •1 ✓

2 marks given

Notes
1 Accept correct use of the cosine rule
2 Treat $\begin{pmatrix}3i\\2j\\0k\end{pmatrix}$ as bad form.

Cosine rule

•1 $\cos(A)=\dfrac{b^2+c^2-a^2}{2bc}$,
and $a=\begin{pmatrix}1\\5\\-4\end{pmatrix}$ *or* $\begin{pmatrix}-1\\-5\\4\end{pmatrix}$
•2 13, 29 *and* 42 and complete

Converse of Pythagoras

•1 *length of third side* $=\sqrt{42}$
•2 $\left(\sqrt{13}\right)^2+\left(\sqrt{29}\right)^2=13+29$
$=42$
$=\left(\sqrt{42}\right)^2$

	Give 1 mark for each •	Illustrations for awarding each •
4	A recurrence relation is defined by $u_{n+1} = pu_n + q$ where $-1 < p < 1$ and $u_0 = 12$. (a) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2** (b) Find the limit of this recurrence relation as $n \to \infty$. **2**	
4	**1.4.3, 1.4.4 CN CB 03/90** **ans: (a)** $p = \frac{1}{3}, q = 11$ **2 marks** **(b)** $16\frac{1}{2}$ **2 marks** •1 ss : e.g. form two equations in p and q •2 pd : process •3 ss : algebraic strategy for limit •4 pd : process limit	•1 $15 = 12p + q, 16 = 15p + q$ •2 $p = \frac{1}{3}, q = 11$ •3 *e.g.* $L = \frac{1}{3}L + 11$ •4 $L = 16\frac{1}{2}$

Example 1

$12 = 16p + q$	•1 ✗
$15 = 15p + q$	
$p = -3, q = 60$	•2 √ **f.t.**
no limit exists since p outside range -1 to 1	•3 √ **f.t.**
	•4 **not available**
	2 marks given

Example 2

$12 = 16p + q$	•1 ✗
$15 = 15p + q$	
$p = -3, q = 60$	•2 √ **f.t.**
$L = \frac{60}{1 - (-3)}$	•3 √
$L = 15$	•4 √
	1 mark given

Notes

1 for •1
either two equations explicitly stated
or
a trial and improvement approach **checking** in particular that u_1 does in fact equal 15 and u_2 does in fact equal 16

2 for (a) correct answers with no working may only earn •2 (one mark being lost through lack of communication)

3 for (a) trial and improvement leading to answers other than the correct ones earn no marks

4 for any rounding eg $p = 0.3$ or 0.33 **instead** of $p = \frac{1}{3}$ in (a) or (b) the candidate loses •2 or •4
BUT candidates may not lose both •2 and •4

6 other acceptable strategies for the limit at •3 are
- $L = \frac{q}{1-p}$
- "lost part" = "add on" i.e. $\frac{2}{3}L = 11$

7 if p has been incorrectly valued ≥ 1 or ≤ -1, •3 may still be awarded for a statement that the limit does not exist but •4 is not available.

8 candidates who choose values for p and q *ex nihilo* may still earn •3 and •4

9 •4 is lost if answers are left like $\frac{11}{\frac{2}{3}}$ but uncancelled fractions e,g $\frac{66}{4}$, are acceptable

	Give 1 mark for each •	Illustrations for awarding each •
5	Given that $f(x)=\sqrt{x}+\frac{2}{x^2}$, find $f'(4)$.	5
5	**1.3.2, 1.3.4 CN C 03/19** **ans : $\frac{3}{16}$** **5 marks** •1 pd : express in standard form •2 pd : express in standard form •3 pd : differentiate fractional index •4 pd : differentiate negative index •5 pd : evaluation	•1 $x^{\frac{1}{2}}$ stated or implied by •3 •2 $2x^{-2}$ stated or implied by •4 •3 $\frac{1}{2}x^{-\frac{1}{2}}$ •4 $-4x^{-3}$ •5 $\frac{3}{16}$

Notes
1 if incorrectly expressed in standard form, follow throughs must match the mark descriptors.
2 •5 can only be awarded on a follow through provided the evaluation involves a fractional index and a negative index.
3 for •5 accept $\frac{12}{64}$.
4 no marks can be gained for finding $f(4)$

Example 1

$f(x)=x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ •1 ✓
•2 ✗
$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}-x^{-\frac{3}{2}}$ •3 √✗ f.t.
•4 √✗ f.t.
$f'(4)=\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$ •5 √✗ f.t.

4 marks given

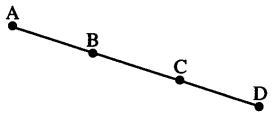
Example 2

$f(x)=x^{-2}+2x^{-2}$ •1 ✗
•2 ✓
$f'(x)=-x^{-3}-4x^{-3}$ •3 √✗ no fractional index
•4 ✓
$f'(4)=-\frac{1}{64}-\frac{1}{16}=-\frac{5}{64}$ •5 √✗ f.t. eased

2 marks given

	Give 1 mark for each •	Illustrations for awarding each •

6 A and B are the points (–1, –3, 2) and (2, –1, 1) respectively.
B and C are the points of trisection of AD, that is AB = BC = CD.
Find the coordinates of D. 3



6 **3.1.6, 3.1.2 CN C 03/48**
ans : (8, 3, –1) 3 marks

- •1 ss : e.g. use a vector approach
- •2 ic : interpret trisection
- •3 pd : process coordinates

- •1 $\vec{AB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ may be stated or implied by •2
- •2 $\vec{AD} = 3\vec{AB} = \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$ may be stated or implied by •3 but not as well as the above!
- •3 $D = (8,3,-1)$

Alternative 1

- •1 $\vec{AB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$
- •2 $C = (5,1,0)$
- •3 $D = (8,3,-1)$

Alternative 2

- •1 $\vec{AB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$
- •2 $\vec{BD} = 2\vec{AB} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}$
- •3 $D = (8,3,-1)$

Alternative 3
one of many forms of the section formula

- •1 $b = \frac{2}{3}a + \frac{1}{3}d$
- •2 *substitution*
- •3 $d = \begin{pmatrix} 8 \\ 3 \\ -1 \end{pmatrix}$

Notes

1 Treat as bad form expressions such as

$$D = \begin{pmatrix} 8 \\ 3 \\ -1 \end{pmatrix} \text{ or } \vec{BD} = (6,4,-2)$$

2 D= (8, 3, –1) with no working may be awarded 2 marks, 1 mark being lost for poor communication

3 A wrong answer with no working earns no marks

4 If A is taken as (2, –1, 1) and B as (–1, –3, 2) then work leading to D(–7, –7, 4) may be awarded 2 marks.

Example 1

C = (5, 1, 0) •1 ✓ •2 ✓
D = (8, 3, –1) •3 ✓

3 marks given

	Give 1 mark for each •	Illustrations for awarding each •

7 Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

7 **2.1.8, 2.1.6** **CN** **B** **03/27**

ans : proof **5 marks**

- •1 ss : substitute linear into quadratic
- •2 pd : express in standard form
- •3 ss : e.g. use discimnant
- •4 pd : evaluate discriminant
- •5 ic : complete proof

- •1 $x^2 + 3x + 4 = 2x + 1$
- •2 $x^2 + x + 3 = 0$ the zero explicitly stated
- •3 $b^2 - 4ac = 1^2$ .....
- •4 $b^2 - 4ac = -11$
- •5 $b^2 - 4ac < 0$ $\therefore$ no intersection

Alternatives for marks •3 and •4

- •3 $a = 1, b = 1, c = 3$
- •4 $b^2 - 4ac = 1 - 4 \times 1 \times 3 < 0$

- •3 $roots = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 3}}{2}$
- •4 $\dfrac{-1 \pm \sqrt{-11}}{2}$

Notes

1 Use of the "alternative" discriminant $b^2 + 4ac$: lose •3 and follow through.
All other versions lose •3, •4 and •5.

- •3 $\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$
- •4 *so* $x^2 + x + 3$ *is* $\cup$, min *at* $\left(-\frac{1}{2}, \frac{11}{4}\right)$

- •3 $\frac{dy}{dx} = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$, $y = \frac{11}{4}$
- •4 *so* $x^2 + x + 3$ *is* $\cup$, min *at* $\left(-\frac{1}{2}, \frac{11}{4}\right)$

- •3 $\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$
- •4 *which is* $\geq \frac{11}{4}$

Example 1

$x^2 + 3x + 4 = 0$
$b^2 - 4ac = 9 - 16$ no marks awarded
etc

Example 2

$x^2 + 3x + 4 \neq 2x + 1$ lose •2 for using the "not equals" sign.
etc Treat the rest as bad form

Example 3

$y = (2x+1)^2 + 3(2x+1) + 4 = 0$ •1 ✗
$4x^2 + 10x + 8 = 0$ •2 ✗
$b^2 - 4ac = 100 - 128 = -28$ •3 ✓
so no intersection •4 ✓
•5 ✓

3 marks given

Example 4

$x^2 + 3x + 4 = 2x + 1$ •1 ✓
$x^2 + x + 3 = 0$ •2 ✓
$b^2 - 4ac < 0$ •3 ✗
so no intersection •4 ✗
•5 ✓

3 marks given

Example 5

$x^2 + 3x + 4 = 2x + 1$ •1 ✓
$x^2 + x + 3 = 0$ •2 ✓
$b^2 - 4ac < 0$ •3 ✗
so no real roots •4 ✗
•5 ✗

2 marks given

	Give 1 mark for each •	Illustrations for awarding each •

8 Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

8 **3.2.3** **CN** **CA 03/55**

ans : $\frac{2}{3}$ **4 marks**

- $\bullet^1$ pd : express in standard form
- $\bullet^2$ pd : integrate
- $\bullet^3$ pd : integrate
- $\bullet^4$ pd : evaluate using limits

- $\bullet^1$ $(3x+1)^{-\frac{1}{2}}$
- $\bullet^2$ $\frac{1}{\frac{1}{2}}(3x+1)^{\frac{1}{2}}$
- $\bullet^3$ $\ldots\times\frac{1}{3}$
- $\bullet^4$ $\frac{2}{3}$

Notes

1 Treat $\frac{2}{3}+c$ as bad form

2 $\frac{1}{1.5}$ does not gain •4

3 •4 is only available after an attempt has been made to integrate

4 •4 is only available if the evaluation involves a fractional power.

Example 1

$\left[\frac{1}{\frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2}\times 3}}\right]_0^1$
$=\frac{9}{2}\left(\frac{1}{8}-1\right)$
$=-\frac{63}{16}$

•1 ✗
•2 ✗
•3 √ f.t.
•4 √ f.t.
2 marks given

Example 2

$\left[\frac{1}{\frac{3}{2}(3x+1)^{-\frac{1}{2}}}\right]_0^1$
$= \ldots$
$=\frac{2}{3}$

•1 ✗
•2 ✗
•3 ✗
•4 √ f.t.
1 mark given

Example 3

$\left[-\frac{3}{2}(3x+1)^{\frac{3}{2}}\right]_0^1$
$= \ldots$
$=\frac{21}{16}$

•1 ✗
•2 ✗
•3 ✗
•4 √ f.t.
1 mark given

Example 4

$\left[2(3x+1)^{\frac{1}{2}}\right]_0^1$
$=2\times 4^{\frac{1}{2}}-2\times 1^{\frac{1}{1}}$
$=2$

•1 ✓
•2 ✓
•3 ✗
•4 √ f.t.
3 marks given

Example 5

$\left[2(3x+1)^{\frac{1}{2}}\right]_0^1$
$= \left[(6x+2)^{\frac{1}{2}}\right]_0^1$
$= \ldots$
$=\sqrt{2}$

•1 ✓
•2 ✓
•3 ✗
•4 ✗
2 marks given

	Give 1 mark for each •	Illustrations for awarding each •

9 Functions $f(x)=\frac{1}{x-4}$ and $g(x)=2x+3$ are defined on suitable domains.

(*a*) Find an expression for $h(x)$ where $h(x)=f(g(x))$. **2**

(*b*) Write down any restriction on the domain of h. **1**

9 **1.2.1, 1.2.6** **CN** **CA 03/5**

(a) ans : $\frac{1}{2x-1}$ **2 marks**

(b) ans : $x\neq\frac{1}{2}$ **1 mark**

$\bullet^1$ ic : start composite function
$\bullet^2$ ic : complete composite function

$\bullet^3$ ic : interpret denominator

$\bullet^1$ $f(2x+3)$ **stated or implied by $\bullet^2$**

$\bullet^2$ $\frac{1}{2x+3-4}$

$\bullet^3$ $x\neq\frac{1}{2}$

Example 1

$\ldots\left(\frac{1}{x-4}\right)$ $\bullet 1$ ✗

$\frac{2}{x-4}+3$ $\bullet 2$ ✓ **f.t.**

$x\neq 4$ $\bullet 3$ ✓ **f.t.**

2 marks given

Notes

1 Use example 1 if candidate finds g(f(x)) [which they may call f(g(x))!!

2 •3 is only available for working containing an algebraic fraction e.g. $\frac{a}{x+c}$ or harder.

3 for •3 accept any statement involving a $\frac{1}{2}$ e.g.

a $x\neq\frac{1}{2}$ (the actual restriction)

b $x=\frac{1}{2}$ (the value to be restricted from the domain)

c $x>\frac{1}{2}$ (part of the restricted domain)

d $x<\frac{1}{2}$ (also part of the restricted domain)

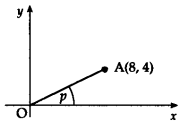
In each case the candidate has identified the value of x which makes the denominator zero (which was the point of (b)).

4 for (b) do not accept unsimplified forms such as $2x-1=0$.

4 for •3, treat $h\neq\frac{1}{2}$ as bad form.

	Give 1 mark for each •	Illustrations for awarding each •

10 A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

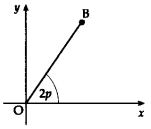


(*a*) Find the exact values of

(i) $\sin(2p)$

(ii) $\cos(2p)$. **5**

The line OB is inclined at an angle $2p$ radians to the x-axis.



(*b*) Write down the exact value of the gradient of OB. **1**

10 **2.3.3, 1.1.6** **NC** **CB 03/34**

(a) ans : $\frac{4}{5}$, $\frac{3}{5}$ **5 marks**

(b) ans : $\frac{4}{3}$ **1 mark**

- $\bullet^1$ pd : calculate hypotenuse
- $\bullet^2$ pd : calculate sinp and cosp
- $\bullet^3$ ss : use double angle formula
- $\bullet^4$ pd : process sin2p
- $\bullet^5$ pd : process cos2p
- $\bullet^6$ pd: relate gradient and tan

- $\bullet^1$ hypot $= \sqrt{80}$
- $\bullet^2$ $\sin(p) = \frac{4}{\sqrt{80}}$ *and* $\cos(p) = \frac{8}{\sqrt{80}}$
- $\bullet^3$ $\sin(2p) = 2\sin(p)\cos(p)$
- $\bullet^4$ $\sin(2p) = \frac{4}{5}$
- $\bullet^5$ $\cos(2p) = \frac{3}{5}$
- $\bullet^6$ $\frac{4}{3}$

Example 1

$\tan(p) = \frac{1}{2}$	•1 ✗
$p = 30°$	•2 ✗
$\sin(p) = \frac{1}{2}$, $\cos(p) = \frac{\sqrt{3}}{2}$	•3 ✓ f.t.
$\sin(2p) = 2\sin(p)\cos(p)$	
$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$	•4 ✓ f.t.
$= \frac{\sqrt{3}}{2}$	
$\cos(2p) = 2\cos^2(p) - 1$	
$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$	
$= \frac{1}{2}$	•5 ✓ f.t.
$\tan(2p) = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$	•6 ✓ f.t.
	3,1 marks given

Example 2

$\tan(p) = \frac{1}{2}$	•1 ✗
$p = 30°$	•2 ✗
$\sin(2p) = \sin 60$	•3 ✗
$= \frac{\sqrt{3}}{2}$	•4 ✗
$\cos(2p) = \cos 60$	•5 ✗
$= \frac{1}{2}$	•6 ✗
$\tan(2p) = \tan 60 = \sqrt{3}$	0 marks given

Notes

1 accept uncancelled fractions for •4, •5 and •6.

e.g. $\frac{64}{80}$, $\frac{48}{80}$ *and* $\frac{64}{48}$ are common

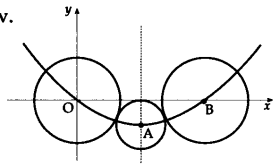
2 marks 4-6 are not available to candidates who base their answers on the assumption that p = 30°, 45 etc so that sin(2p) = sin(60) etc. See examples 1 & 2.

Example 3

a wrong hypotenuse leading to	•1 ✗
$hyp = \sqrt{32}$	•2 ✓
$\sin(p) = \frac{4}{\sqrt{32}}$, $\cos(p) = \frac{8}{\sqrt{32}}$	•3 ✓
$\sin(2p) = 2\sin(p)\cos(p)$	
$= 2 \times \frac{4}{\sqrt{32}} \times \frac{8}{\sqrt{32}}$	•4 ✗
$= 2$	
$\cos(2p) = 2\cos^2(p) - 1$	
$= 2\left(\frac{8}{\sqrt{32}}\right)^2 - 1$	
$= \frac{3}{2}$	•5 ✓
$\tan(2p) = \frac{2}{\frac{3}{2}} = \frac{4}{3}$	•6 ✓
	3,1 marks given

	Give 1 mark for each •	Illustrations for awarding each •

11 • O, A and B are the centres of the three circles shown in the diagram below.
• The two outer circles are congruent and each touches the smallest circle.
• Circle centre A has equation $(x-12)^2+(y+5)^2=25$.
• The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y=px(x+q)$.
Find the values of p and q. **2**

11 **2.4.1, 2.1.10 CN CBA 03/40**

(a) ans : A(12, –5), OA = 13

$(x-24)^2+y^2=64$ **5 marks**

(b) ans : $p=\frac{5}{144}$, $q=-24$ **2 marks**

•1 ic : interpret centre
•2 pd : use Pythagoras
•3 ic : interpret radius
•4 ic : interpret centre
•5 ic : state equ of circle
•6 pd : process
•7 pd : process

•1 $A=(12,-5)$
•2 $OA=13$ accept $\sqrt{169}$
•3 $r_B=8$ **stated or implied by •5**
•4 $B=(24,0)$ **stated or implied by •5**
•5 $(x-24)^2+y^2=64$
•6 $p=\frac{5}{144}$
•7 $q=-24$

Notes
1 Take care with the implications at •3 and •4. Only the correct values for r and B can be implied by •5. Incorrect values of r and/or B must be stated before the equation of the circle is given in order that •5 can be awarded as a follow-through mark.

	Give 1 mark for each •	Illustrations for awarding each •

12 Simplify $3\log_e(2e) - 2\log_e(3e)$, expressing your answer in the form $A + \log_e(B) - \log_e(C)$ where A, B and C are whole numbers. **4**

12 **3.3.6, 3.3.2** **CN** **BA 03/43**

ans : 1 + ln(8) - ln(9) **4 marks**

•1 pd : use log laws
•2 pd : use log laws
•3 pd : process
•4 pd : use log laws

•1 $\ln(2e)^3 - \ln(3e)^2$
•2 $\ln\left(\frac{(2e)^3}{(3e)^2}\right)$
•3 $\ln\left(\frac{8e}{9}\right)$
•4 $1 + \ln(8) - \ln(9)$

Alternative 1

•1 $3[\ln(2) + \ln(e)]$
•2 $-2[\ln(3) + \ln(e)]$
•3 $3\ln(2) + 3 - 2\ln(3) - 2$
•4 $1 + \ln(8) - \ln(9)$

Notes

1 $\ln 2e^3 - \ln 3e^2$ will not gain •1 unless you see an '8' and a '9' appearing in subsequent work, in which case you can treat it as bad form.

Example 1

$\ln(2e)^3 - \ln(3e)^2$ •1 ✓
$\ln\left(\frac{2e^3}{3e^2}\right)$ •2 ✗
$\ln\left(\frac{2e}{3}\right)$ •3 √✗
$1 + \ln(2) - \ln(3)$ •4 √✗
3 marks given

Example 2

$\ln(2e)^3 - \ln(3e)^2$ •1 ✓
$\ln(8e) - \ln(9e)$ •3 ✗
$\ln\left(\frac{8e}{9e}\right)$ •2 √✗
$\ln(8) - \ln(9)$ •4 √✗ **Eased**
2 marks given

Example 3

$\ln(2e)^3 - \ln(3e)^2$ •1 ✓
$\ln\left(\frac{2e^3}{3e^2}\right)$ •2 ✓ ev. line 3
$\ln\left(\frac{8}{9}\right)$ •3 ✗
$\ln(8) - \ln(9)$ •4 √✗ **Eased**
2 marks given

Example 4

$\ln 2e^3 - \ln 3e^2$ •1 ✓ see line 2
$\ln\left(\frac{8e^3}{9e^2}\right)$ •2 ✓
•3 ✓ ev in line 4
$\ln(8) - \ln(9) + \ln(e)$ •4 √✗
3 marks given

Example 5

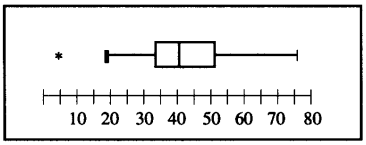
$\ln(8e) - \ln(9e)$ •1 ✗
$\ln\left(\frac{8e}{9}\right)$ •2 ✗
•3 ✗
$1 + \ln(8) - \ln(9)$ •4 √✗
1 mark given

Example 6

$\ln(8e) - \ln(9e)$ •1 ✗
$\ln\left(\frac{8e}{9e}\right)$ •2 √✗
•3 √✗ **Eased**
$\ln(8) - \ln(9)$ •4 √✗ **Eased**
1 mark given

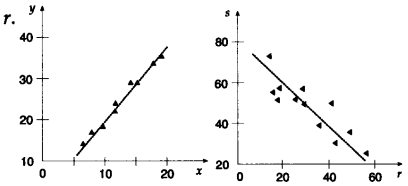
Example 7

$3\ln(2) + \ln(e) - 2\ln(3) + \ln(e)$ •1 ✗
$\ln(8) + 1 - \ln(9) + 1$ •2 ✗
$2 + \ln(8) - \ln(9)$ •3 √✗
•4 √✗
2 marks given

	Give 1 mark for each •	Illustrations for awarding each •
1	The times taken by a group of students to complete a statistical project are given in the stem-and-leaf diagram. Identify any outliers and illustrate the data with a box-plot.	Time taken to the nearest minute 0 5 1 9 2 7 7 8 3 0 2 4 5 5 6 8 9 4 0 0 1 2 2 3 6 8 8 9 5 1 3 3 4 8 8 6 0 7 6 n = 31 5 | 3 represents 53 minutes **5**
S1	**4.1.2, 4.1.4 CN C 03/64** **ans : 5 is outlier, boxplot 5 marks** •1 pd : calculate quartiles •2 ss : know how to caculate fence •3 ic : determine upper fence/outliers •4 ic : determine lower fence/outliers •5 ic : determine mean/draw box-plot	•1 34 & 51 •2 $LF = Q_1 - \frac{3}{2}(Q_3 - Q_1)$ •3 76·5 & no outliers •4 8.5 & 5 min. is outlier •5 41 & box – plot
		

	Give 1 mark for each •	Illustrations for awarding each •

2 The diagrams below show the scattergraphs of y on x and s on r.



The equation of the least squares regression line of y on x is $y = 1{\cdot}7x + 2$.

The equation of the least squares regression line of s on r is $s = 81 - 1{\cdot}05r$.

(a) Predict the expected value of (i) y when $x = 10$ **2**

(ii) s when $r = 20$.

(b) Which prediction is more reliable? Give a reason for your answer. **1**

S2	**4.4.3** **NC C 03/59** **(a) ans : 19, 60** **2 marks** **(b) ans : y on x more reliable** •[1] pd : use regression equation •[2] pd : use regresion equation •[3] ic : interpret scatter diagram	•[1] 19 •[2] 60 •[3] y on x more reliable as diagram shows stronger relationship

	Give 1 mark for each •	Illustrations for awarding each •

3 A farmer sells eggs in boxes of 6. The discrete random variable X represents the number of brown eggs in a box.

X has the following probability distribution:

$$P(X=x)=\begin{cases}\frac{1}{3}k(7-x) & \text{for } x = 0,1,2,3,4,5 \text{ and } 6\\ 0 & \text{otherwise}\end{cases}$$

where k is a constant.

(*a*) Find the value of k. **2**

(*b*) Find the expected value and variance of X, the number of brown eggs in a box. **3**

S3 **4.2.11, 4.2.12** **CN** **C 03/68**

(a) ans : $\frac{3}{28}$ **2 marks**

(b) ans : 2, 3 **3 marks**

- •1 ss : use $\Sigma P(X)=1$
- •2 pd : evaluate k
- •3 pd : calculate expected value
- •4 pd : calculate $E(X^2)$
- •5 pd : calculate variance

- •1 $P(X)$ $\frac{7k}{3}, \frac{6k}{3}, \frac{5k}{3}, \frac{4k}{3}, \frac{3k}{3}, \frac{2k}{3}, \frac{k}{3}$
- •2 $\Sigma P(X)=1 \Rightarrow k=\frac{3}{28}$
- •3 $E(X)=2$
- •4 $E(X^2)=7$
- •5 $V(X)=3$

x	0	1	2	3	4	5	6	
$P(x)$	$\frac{7k}{3}$	$\frac{6k}{32}$	$\frac{5k}{3}$	$\frac{4k}{3}$	$\frac{3k}{3}$	$\frac{2k}{3}$	$\frac{k}{3}$	$\Sigma = \frac{28k}{3}=1 \quad k=\frac{3}{28}$
$P(x)$	7	6	5	4	3	2	1	$\times\frac{1}{28}$
$xP(x)$	0	6	10	12	12	10	6	$\times\frac{1}{28}$ $\quad \Sigma = \frac{56}{28}=2$
$x^2P(x)$	0	6	20	36	48	50	36	$\times\frac{1}{28}$ $\quad \Sigma = \frac{196}{28}=7$
								$\text{var}=7-2^2=3$

Give 1 mark for each •	Illustrations for awarding each •
Additional marks in Paper 1	
Question 1 +1	
•1 ic : rearrange in standard form •2 ic : interpret gradient from linear equ. •3 ic : find perp. gradient •4 ic : state equation of line	•1 $y=-4x+1$ •2 $m=-4$ •3 $m_{perp}=\frac{1}{4}$ •4 $y-3=\frac{1}{4}(x-(-1))$
Question 2 +1	
•1 ic : start to complete square •2 pd : finish completing the square •3 ic : sketch •4 ic : sketch •5 ic : sketch	•1 $(x+3)^2$ •2 $+2$ •3 *U – shaped parabola* •4 *minimum at* $(-3,2)$ •5 *intercept on* $y-$*axis at* $(0,11)$
Question 3 +1	
•1 ic : interpret unit vectors •2 ss : know to use scalar product and get zero •3 pd : process	•1 $\begin{pmatrix}3\\2\\0\end{pmatrix}$ *and* $\begin{pmatrix}2\\-3\\4\end{pmatrix}$ •2 for perpendicularity "$u.v$"$=0$ •3 $\begin{pmatrix}3\\2\\0\end{pmatrix}\begin{pmatrix}2\\-3\\4\end{pmatrix}=6-6+0=0$
Question 4 +1	
•1 ss : e.g. form two equations in p and q •2 pd : process •3 ic : state the condition for limit to exist •4 ss : algebraic strategy for limit •5 pd : process limit	•1 $15=12p+q, 16=15p+q$ •2 $p=\frac{1}{3}, q=11$ •3 since $-1<\frac{1}{3}<1$, limit exists •4 *e.g.* $L=\frac{1}{3}L+11$ •5 $L=16\frac{1}{2}$
Question 5 +1	
•1 pd : express in standard form •2 pd : express in standard form •3 pd : differentiate fractional index •4 pd : differentiate negative index •5 pd : evaluation •6 pd : evaluation	•1 $x^{\frac{1}{2}}$ •2 $2x^{-2}$ •3 $\frac{1}{2}x^{-\frac{1}{2}}$ •4 $-4x^{-3}$ •5 $\frac{1}{2}\times4^{-\frac{1}{2}}=\frac{1}{4}$ *or* $-4\times4^{-3}=-\frac{1}{16}$ •6 $\frac{3}{16}$

Give 1 mark for each •	Illustrations for awarding each •
Question 8 +1	
•1 pd : express in standard form	•1 $(3x+1)^{-\frac{1}{2}}$
•2 pd : integrate	•2 $\frac{1}{\frac{1}{2}}(3x+1)^{\frac{1}{2}}$
•3 pd : integrate	•3 $\ldots\times\frac{1}{3}$
•4 ic : substitute the limits	•4 $\left[\frac{2}{3}(3\times1+1)^{\frac{1}{2}}\right]-\left[\frac{2}{3}(3\times0+1)^{\frac{1}{2}}\right]$
•5 pd : evaluate	•5 $\frac{2}{3}$
Question 10 +2	
	•1 hypot $=\sqrt{80}$
•1 pd : calculate hypotenuse	•2 $\sin(p)=\frac{4}{\sqrt{80}}$ *and* $\cos(p)=\frac{8}{\sqrt{80}}$
•2 pd : calculate sinp and cosp	•3 $\sin(2p)=2\sin(p)\cos(p)$
•3 ss : use double angle formula	•4 $\sin(2p)=\frac{4}{5}$
•4 pd : process sin2p	•5 $\cos(2p)=2\cos^2(p)-1$
•5 ss : use double formula	•6 $\cos(2p)=\frac{3}{5}$
•6 pd : process cos2p	
	•7 gradient = $\tan(2p)$
•7 ic: relate gradient and tan	•8 $\frac{4}{3}$
•8 pd : process	
Question 11 +1	
	•1 $A=(12,-5)$
•1 ic : interpret centre	•2 $OA=13$
•2 pd : use Pythagoras	•3 $r_B=8$
•3 ic : interpret radius	•4 $B=(24,0)$
•4 ic : interpret centre	•5 $(x-24)^2+y^2=64$
•5 ic : state equ of circle	•6 $q=-24$
•6 ic : interpret B and q	•7 substitute $(12,-5)$
•7 ss : strategy for p	•8 $p=\frac{5}{144}$
•8 pd : process	

Increase in marks for Paper 1 = 9
Increase in marks for Paper 2 = 11
Total increase in marks = 20.

For 2004 the marks will allocated as follows:
Paper 1 60
Paper 2 70
Total 130

[END OF MARKING INSTRUCTIONS]

