

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

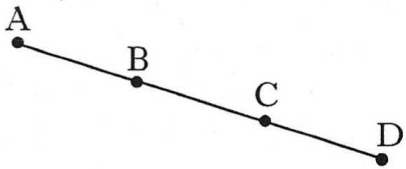
$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

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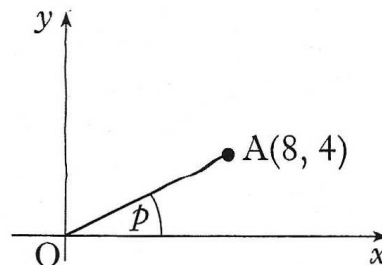
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

1. Find the equation of the line which passes through the point $(-1, 3)$ and is perpendicular to the line with equation $4x + y - 1 = 0$. 3
2. (a) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. 2
 (b) Hence or otherwise sketch the graph of $y = f(x)$. 2
3. Vectors \mathbf{u} and \mathbf{v} are defined by $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.
 Determine whether or not \mathbf{u} and \mathbf{v} are perpendicular to each other. 2
4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.
 (a) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q . 2
 (b) Find the limit of this recurrence relation as $n \rightarrow \infty$. 2
5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. 5
6. A and B are the points $(-1, -3, 2)$ and $(2, -1, 1)$ respectively.
 B and C are the points of trisection of AD, that is $AB = BC = CD$.
 Find the coordinates of D. 3
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7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. 5
8. Find $\int_0^1 \frac{dx}{(3x + 1)^{\frac{1}{2}}}$. 4
9. Functions $f(x) = \frac{1}{x - 4}$ and $g(x) = 2x + 3$ are defined on suitable domains.
 (a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. 2
 (b) Write down any restriction on the domain of h . 1

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x -axis.

(a) Find the exact values of:

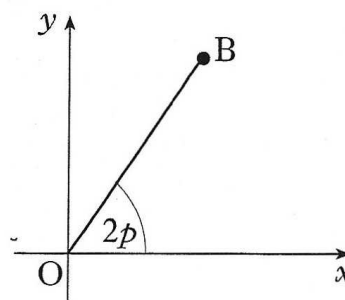
- (i) $\sin(2p)$;
(ii) $\cos(2p)$.



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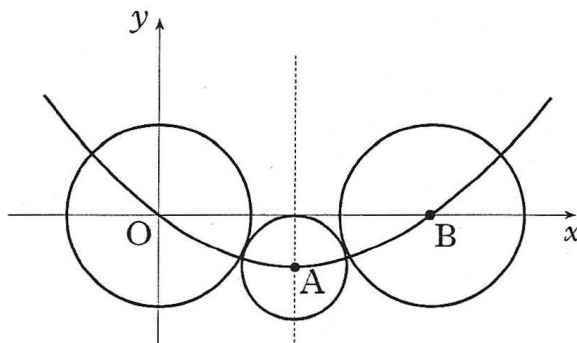
The line OB is inclined at an angle $2p$ radians to the x -axis.

(b) Write down the exact value of the gradient of OB.



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11. • O, A and B are the centres of the three circles shown in the diagram below.
• The two outer circles are congruent and each touches the smallest circle.
• Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
• The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



- (a) (i) State the coordinates of A and find the length of the line OA. 2
(ii) Hence find the equation of the circle with centre B. 3
- (b) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of p and q . 2

12. Simplify $3 \log_e(2e) - 2 \log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. 4