FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a.b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(\mathrm{A} \pm \mathrm{B}) &= \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B} \\
\cos(\mathrm{A} \pm \mathrm{B}) &= \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B} \\
\sin 2\mathrm{A} &= 2\sin \mathrm{A} \cos \mathrm{A} \\
\cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\
&= 2\cos^2 \mathrm{A} - 1 \\
&= 1 - 2\sin^2 \mathrm{A}
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

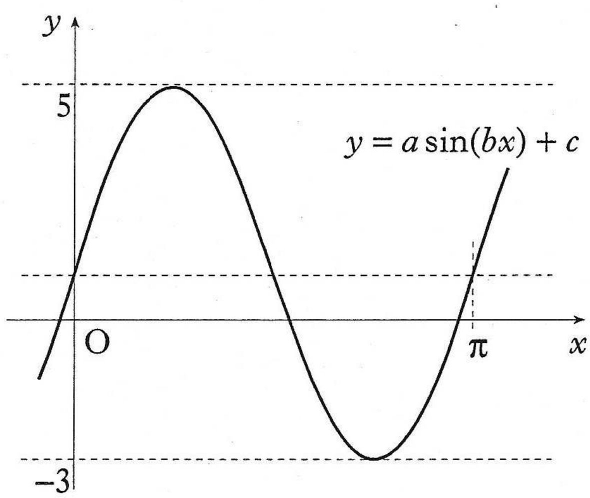
1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

(a) Show that $(x - 2)$ is a factor of $f(x)$.

(b) Express $f(x)$ in its fully factorised form. 4

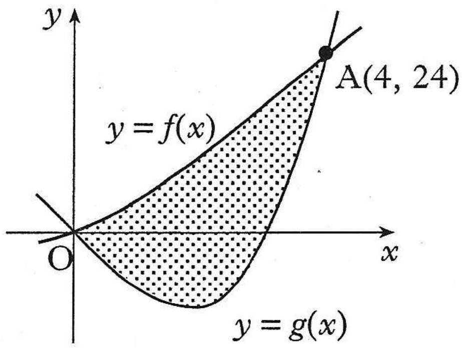
2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

Determine the values of a, b and c. 3



3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves. 5

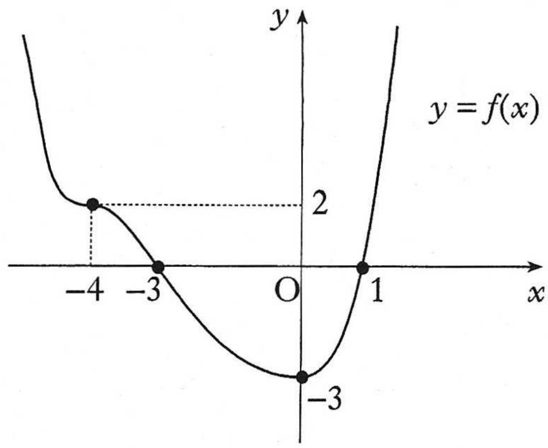


4. (a) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. 5

(b) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. 6

5. The diagram shows the graph of a function f.

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

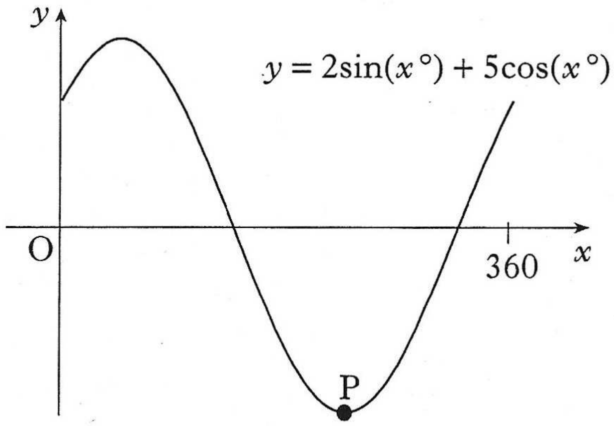


(a) Sketch the graph of $y = f(-x)$. **2**

(b) On the same diagram, sketch the graph of $y = 2f(-x)$. **2**

6. If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

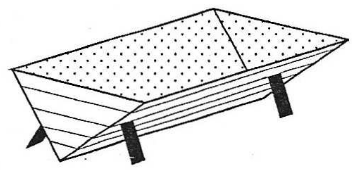
7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.



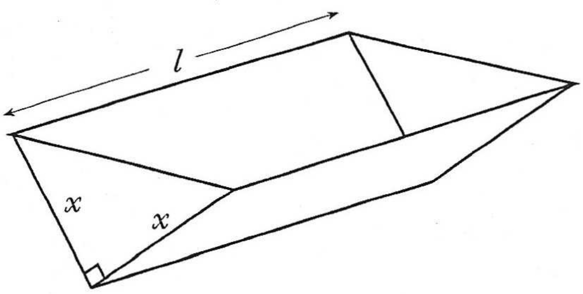
(a) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a < 360$. **4**

(b) Find the coordinates of the minimum turning point P. **3**

8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



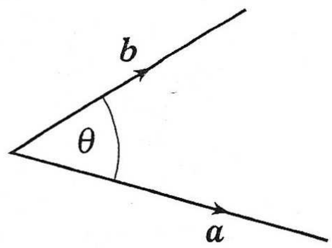
The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



(a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**

(b) Find the value of x which minimises this surface area. **5**

9. The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. **4**



10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places. **5**

11. (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.

(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**