SCOTTISH
QUALIFICATIONS
AUTHORITY

2005 Mathematics

Higher

Finalised Marking Instructions

These Marking Instructions have been prepared by Examination Teams for use by SQA Appointed Markers when marking External Course Assessments.

Mathematics Higher

Instructions to Markers

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of marks(s) should be made.

   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✓̸ or ✗ ✓). In appropriate cases attention may be directed to work which is not quite correct (eg bad form) but which has not been penalised, by underlining with a dotted or wavy line.

   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✓̸̸).

5.
   - The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   - Only the mark should be written, **not** a fraction of the possible marks.
   - These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.

   Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:

- working subsequent to a correct answer
- legitimate variations in numerical answers
- correct working in the "wrong" part of the question
- omission of units
- bad form

9. No piece of work should be scored through without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme – answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the PA. Please see the general instructions for PA referrals.

12. No marks should be deducted at this stange for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.

14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.

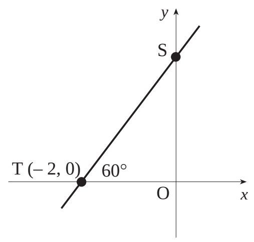
15. **Do not write any comments on the scripts.** A **revised** summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. **Tick** correct working.
2. Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3. Do **not** write marks as fractions.
4. Put each mark **at the end** of the candidate's response to the question.
5. **Follow through** errors to see if candidates can score marks subsequent to the error.
6. Do **not** write any comments on the scripts.

1 Find the equation of the line ST, where T is the point (– 2, 0) and angle STO is 60°. **3**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1		3	C	G2, G3	NC	05/6

The primary method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- •1 ss use $m = \tan\theta$
- •2 pd use exact value
- •3 ic interpret result

Primary Method : Give 1 mark for each •

- •1 $m = \tan\left(60°\right)$ **stated or implied by •2**
- •2 $m = \sqrt{3}$
- •3 $y - 0 = \sqrt{3}\left(x - (-2)\right)$

3 marks

Notes

1 A candidate who states $m = \tan(\theta°)$, and does not go on to use it earns no marks.

Incompletion 1

$m = \tan(60°)$
$y - 0 = \tan(60°)(x - (-2))$
•1 ×√
•2 ×
•3 ×√

award 2 marks

Common Error 1

$m = \sin(60°)$
$y - 0 = \frac{\sqrt{3}}{2}(x - (-2))$
•1 ×
•2 ×√
•3 ×√

award 2 marks

Alternative Method 1

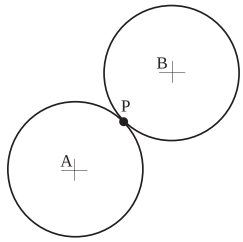
- •1 $\text{OS} = 2\tan(60°) = 2\sqrt{3}$
- •2 $m = \frac{2\sqrt{3}}{2} = \sqrt{3}$ $\left(cf\ \ y = mx + c\right)$
- •3 $y = \sqrt{3}x + 2\sqrt{3}$

Alternative Method 2

- •1 $\cos(60°) = \frac{2}{ST}$ *leading to* $ST = 4$ *and* $OS = \sqrt{12}$
- •2 $m = \frac{\sqrt{12}}{2}$
- •3 $y - 0 = \frac{\sqrt{12}}{2}\left(x - (-2)\right)$

2 Two congruent circles, with centres A and B, touch at P.
Relative to suitable axes, their equations are

$x^2 + y^2 + 6x + 4y - 12 = 0$ and $x^2 + y^2 - 6x - 12y + 20 = 0$.



(*a*) Find the coordinates of P. 3

(*b*) Find the length of AB. 2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	3	C	G9, G6	CN	05/18
	b	2	C	G9	CN	

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- •1 ic interpret equ. of circle
- •2 ic interpret equ. of circle
- •3 pd process midpoint
- •4 ss know how to find length
- •5 pd process

Primary Method : Give 1 mark for each •

- •1 centre $A = (-3,-2)$ **[Note 1]**
- •2 centre $B = (3,6)$
- •3 $P = (0,2)$

3 marks

- •4 $AB^2 = \left(3-(-3)\right)^2 + \left(6-(-2)\right)^2$ **[CE 1]**
- •5 $AB = 10$ **[Note 2]**

2 marks

Notes

1 at **•1, •2**
Each of the following may be awarded 1 mark from the first two marks

$A = (6,4)$ *and* $B = (-6,-12)$

$A = (-6,-4)$ *and* $B = (6,12)$

$A = (3,2)$ *and* $B = (-3,-6)$

2 At **•5** stage, some errors lead to unsimplified surds.
DO NOT accept unsimplified square roots of perfect squares (up to 100).
e.g. √100 would not gain •5.

Common Error 1 for (b)

$AB^2 = \left(3+(-3)\right)^2 + \left(6+(-2)\right)^2$

$AB = 4$

•4 ×

•5 ×√

award 1 mark for (b)

Alternative Method 1 for marks 1,2,3

$\boldsymbol{p} = \frac{1}{2}(\boldsymbol{b}+\boldsymbol{a})$

•1 $\boldsymbol{b} = \begin{pmatrix}3\\6\end{pmatrix}$

•2 $\boldsymbol{a} = \begin{pmatrix}-3\\-2\end{pmatrix}$

•3 $P = (0,2)$ **[Note 1]**

Notes

1 Treat $P = \begin{pmatrix}0\\2\end{pmatrix}$ as bad form.

Alternative Method 2 for marks 4,5

•4 $r^2 = 3^2 + 2^2 - (-12)$
or $r^2 = (-3)^2 + (-6)^2 - 20$

•5 $AB = 2r = 10$

Alternative Method 3 for marks 4,5

•4 $\overrightarrow{AB} = \begin{pmatrix}6\\8\end{pmatrix}$

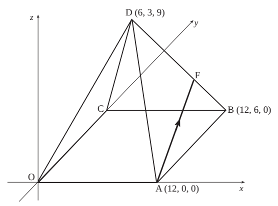
•5 $AB = 10$

6

3 D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).
F divides DB in the ratio 2 : 1.

(*a*) Find the coordinates of the point F. **4**

(*b*) Express $\vec{AF}$ in component form. **1**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	4	C	G25	CN	05/24
	b	1	C	G17	CN	

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- •1 ss know to find $\overrightarrow{DB}$
- •2 ic interpret ratio
- •3 pd process scalar times vector
- •4 ic interpret vector and end points
- •5 ic interpret coordinates to vector

Primary Method : Give 1 mark for each •

•1 $\overrightarrow{DB} = \begin{pmatrix} 12-6 \\ 6-3 \\ 0-9 \end{pmatrix}$

•2 $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DB}$

•3 $\overrightarrow{DF} = \frac{2}{3}\begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$

•4 $\mathrm{D} = (6,3,9)$ *so* $\mathrm{F} = (10,5,3)$ ***[Note 1]*** **4 marks**

•5 $\overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$ **1 mark**

Notes

1 Do not penalise candidates who write the coordinates of F as a column vector (treat as bad form).

2 A correct answer to (a) with no working may be awarded one mark only.

3 For guessing the coordinates of F, no marks should be awarded in (a).
1 mark is still available in (b) provided the guess in (a) is geographically compatible with the diagram
ie $0 \le x \le 12$
$3 \le y \le 6$
$0 \le z \le 9$

4 In (a)
Where the ratio has been reversed (ie 1:2) leading to F=(8, 4, 6) then 3 marks may be awarded (•1, •3, •4).

5 In (b)
Accept $\overrightarrow{AF} = -2\boldsymbol{i} + 5\boldsymbol{j} + 3\boldsymbol{k}$ for **•5**.

Alternative Method 1 [Marks 1-4]

•1 $\overrightarrow{DF} = 2\overrightarrow{FB}$ s/i by •2

•2 $\boldsymbol{f} - \boldsymbol{d} = 2\boldsymbol{b} - 2\boldsymbol{f}$

•3 $3\boldsymbol{f} = 2\begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}$

•4 $\mathrm{F} = (10,5,3)$ ***[Note 1]***

Alternative Method 2 [Marks 1-4]

•1 $\boldsymbol{f} = \frac{m\boldsymbol{b} + n\boldsymbol{d}}{m+n}$ s/i by •3

•2 $m = 2, n = 1$ s/i by •3

•3 $\boldsymbol{f} = \frac{1}{3}\left(2\begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}\right)$

•4 $\mathrm{F} = (10,5,3)$ ***[Note 1]***

Alternative Method 3 [Marks 1-5]

•1 $\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$

•2 $\overrightarrow{AF} = \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BD}$

•3 $\overrightarrow{AF} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \frac{1}{3}\left[\begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix}\right]$

•4 $\overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$

•5 $\left(\mathrm{A} = (12,0,0 \text{ so}\right)$ $\mathrm{F} = (10,5,3)$

Alternative Method 4 [Marks 1-4]

x	6	•	10	12	•1
y	3	•	5	6	•2
z	9	•	3	0	•3

so F = (10, 5, 3) •4

7

4 Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

(*a*) Find $h(x)$ where $h(x) = g(f(x))$. **2**

(*b*) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.

(ii) Hence state the range of the function h. **2**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	2	C	A4	NC	05/7
	b	2	C	A1	NC	

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- •1 ic interpret comp. function build-up
- •2 ic interpret comp. function build-up
- •3 ic interpret function
- •4 ic interpret function

Primary Method : Give 1 mark for each •

- •1 $g(3x-1)$ ***stated or implied by •2***
- •2 $(3x-1)^2 + 7$ **2 marks**
- •3 $\left(\frac{1}{3}, 7\right)$ ***[Note 1]***
- •4 $y \geq 7$ ***[Note 2]*** **2 marks**

Notes

1 For **•3**
No justification is required for **•3**. Candidates may choose to dfferentiate etc but may still only earn one mark for a correct answer.

2 For **•4**
Accept $y > 7,\ h \geq 7,\ h > 7,\ h(x) > 7,\ h(x) \geq 7$
Do not accept $x \geq 7,\ x > 7$

Common Error No.1

- •1 × $f(x^2 + 7)$
- •2 ×√ $3x^2 + 20$
- •3 ×√ $(0, 20)$
- •4 ×√ $y \geq 20$

award 3 marks

Notes 1 & 2 apply.

5	Differentiate $(1+2\sin(x))^4$ with respect to x.	2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		2	A	C20, C21	CN	05/28

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$\bullet^1$ pd start differentiation process

$\bullet^2$ pd use the chain rule

Primary Method : Give 1 mark for each •

$\bullet^1$ $4(1+2\sin(x))^3$

$\bullet^2$ ... $\times 2\cos(x)$

2 marks

Common Error 1

$\bullet^1$ × $1+2\sin^4(x)$

$\bullet^2$ ×√ $8\sin^3(x)\times\cos(x)$

award 1 mark

Common Error 2

$\bullet^1$ × $1+16\sin^4(x)$

$\bullet^2$ ×√ $64\sin^3(x)\times\cos(x)$

award 1 mark

Common Error 3
[mixture of differentiating and integrating]

$\bullet^1$ × $\frac{1}{4}\left(1+2\sin(x)\right)^3$

$\bullet^2$ × $\times\frac{1}{2}\cos(x)$

award 0 marks

Common Error 4

$\bullet^1$ × $4\left(1+2\sin(x)\right)^5$

$\bullet^2$ ×√ $\times 2\cos(x)$

award 1 mark

6 (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. **2**

(*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. **5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6	a	2	C	A13	CN	05/42
	b	5	B	A11, A13	CN	

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- •1 ss know how to find limit
- •2 pd process
- •3 ic interpret rec. relation
- •4 ic interpret rec. relation
- •5 pd arrange in standard form
- •6 pd process a quadratic
- •7 ic use limit condition

Primary Method : Give 1 mark for each •

- •1 $e.g.\ 4 = k \times 4 + 5$ *[Notes 1,2,3]*
- •2 $k = -\frac{1}{4}$ **2 marks**
- •3 $u_1 = 3m + 5$
- •4 $u_2 = m(3m+5) + 5$ *[Note 4]*
  $\left(m(3m+5)+5 = 7\right)$
- •5 $3m^2 + 5m - 2 = 0$ *[Note 5]*
- •6 $(3m-1)(m+2) = 0$
- •7 $m = -2$ **5 marks**

Alternative Method 1 for (a)

Using $L = \dfrac{b}{1-a}$

- •1 $4 = \dfrac{5}{1-k}$
- •2 $k = -\dfrac{1}{4}$

Alternative Method 2 for (a)

$L = kL + 5$

$kL = L - 5$

- •1 $k = \dfrac{L-5}{L}$
- •2 $k = \dfrac{4-5}{4} = -\dfrac{1}{4}$

Notes
for (a)

1 Guess and Check
Guessing $k = -0{\cdot}25$ and checking algebraically or iteratively that this does yield a limit of 4 may be awarded 1 mark.

2 No working
Simply stating that $k = -0{\cdot}25$ earns no marks.

3 Wrong formula
Work using an incorrect 'formula' leading to a valid value of k (ie $|k|<1$) may be awarded 1 mark.

for (b)

4 If u_2 is not a quadratic, then no further marks are available.

5 An "=0" must appear at least once in working at the •5/•6 stage.

6 For candidates who make errors leading to no values outside the range $-1 < m < 1$, or to two values outside the range, then they must say why they are accepting or rejecting in order to gain •7

7 For **•7**, either crossing out the "1/3" or underlining the "–2" is the absolute minimum communication required for this i/c mark. [A statement would be preferable]

Common Error 1

- •1 × $4 = \dfrac{5}{1-a}$
- •2 ×√ $a = -\dfrac{1}{4}$

award 1 mark

Common Error 2

- •3 √ $u_1 = 3m + 5$
- •4 × $u_2 = 3m^2 + 5$
- •5 × $3m^2 = 2$ *or equivalent*
- •6 × $m = \sqrt{\frac{2}{3}}$ *(eased)*
- •7 ×√ there are no values which do not yield a limit

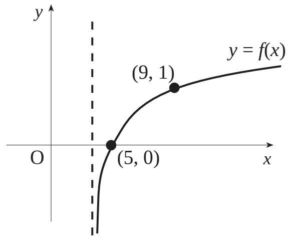
award 2 marks

7 The function f is of the form $f(x) = \log_b(x - a)$.

The graph of $y = f(x)$ is shown in the diagram.

(a) Write down the values of a and b. **2**

(b) State the domain of f. **1**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7	a	2	C	A7	NC	05/9
	b	1	C	A1	NC	

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- $\bullet^1$ ic interpret the translation
- $\bullet^2$ ic interpret the base
- $\bullet^3$ ic interpret diagram

Primary Method : Give 1 mark for each •

- $\bullet^1$ $a = 4$
- $\bullet^2$ $b = 5$ ***[Note 1]*** **2 marks**
- $\bullet^3$ domain is $x > a$ ***[Note 2]*** **1 mark**

Notes

1 No justification is required for marks 1 and 2.
BUT simply stating

$$0 = \log_b(5 - a) \textbf{ and } 1 = \log_b(9 - a)$$

with no further work earns no marks.

However

$$1 = \log_b(9 - a) \textbf{ and } b = 9 - a$$

may be awarded 1 mark.
Of course to gain the other mark, both values would need to be stated.

2 Clearly $x > 4$ is correct

but **do not** accept a domain of $x \geq 4$.

8 A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(*c*) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. **5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	5	C	A21	NC	05/10
	b	2	C	A21	NC	
	c	5	B	C11	NC	

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- •1 ss know to use $x = 3$
- •2 pd complete strategy
- •3 ic interpret zero remainder
- •4 ic interpret quadratic factor
- •5 pd complete factorising

Primary Method : Give 1 mark for each •

•1 *eg*
$$3 \;\Big|\; 2 \quad -7 \quad 0 \quad 9$$

•2 *eg*
$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 9 \\ & & 6 & -3 & -9 \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

•3 remainder is zero so $(x - 3)$ is a factor ***[Note 1]***

•4 $2x^2 - x - 3$

•5 $(x - 3)(2x - 3)(x + 1)$ **stated explicitly** **5 marks**

Notes

In the Primary method, **(a)**

1 Candidates must show some acknowledgement of the result of the synthetic division. Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.

2 Candidates may use a second synthetic division to complete the factorisation. •4 and •5 are available.

Alternative method 1 (marks 1-5) (linear factor by substitution)

•1 $f(3) = \ldots$

•2 $f(3) = 2 \times 3^3 - 7 \times 3^2 + 9 = 54 - 63 + 9 = 0$

•3 *eg*
$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 9 \\ & & 6 & & \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

•4 $2x^2 - x - 3$

•5 $(x - 3)(2x - 3)(x + 1)$

Alternative method 3 (marks 1-5) (quad factor by inspection)

•1 $f(3) = \ldots$

•2 $f(3) = 2 \times 3^3 - 7 \times 3^2 + 9 = 54 - 63 + 9 = 0$

•3 $(x - 3)(2x^2 \;\ldots\ldots\ldots)$

•4 $(x - 3)\left(2x^2 - x - 3\right)$

•5 $(x - 3)(2x - 3)(x + 1)$

Alternative method 2 (marks 1-5) (long division)

•1
$$\begin{array}{r|llll} & 2x^2 & & & \\ \hline x - 3 & 2x^3 & -7x^2 & & +9 \\ & 2x^3 & -6x^2 & & \\ \hline & & -x^2 & & \end{array}$$

•2
$$\begin{array}{r|llll} & 2x^2 & -x & -3 & \\ \hline x - 3 & 2x^3 & -7x^2 & & +9 \\ & 2x^3 & -6x^2 & & \\ \hline & & -x^2 & & \\ & & -x^2 & +3x & \\ \hline & & & -3x & +9 \\ & & & -3x & +9 \\ \hline \end{array}$$

•3 remainder is zero so $(x - 3)$ is a factor

•4 $(x - 3)\left(2x^2 - x - 3\right)$

•5 $(x - 3)(2x - 3)(x + 1)$

12

8	A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.	
	(a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.	5
	(b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes.	2
	(c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.	5

•6	ic	interpret y-intercept
•7	ic	interpret x-intercepts
•8	ss	set derivative to zero
•9	pd	solve
•10	ss	evaluate function at an end point
•11	ic	interpret results
•12	ic	interpret results

Primary Method : Give 1 mark for each •

•6	$(0,9)$		
•7	$(-1,0), \left(\frac{3}{2},0\right), (3,0)$	***[Note 3]***	**2 marks**
•8	$6x^2 - 14x = 0$		
•9	$x = 0$ *or* $x = \frac{14}{6}$	***[Note 6]***	
•10	$f(-2) = -35$ ***OR*** $f(2) = -3$		
•11	*greatest value* $= 9$		
•12	*least value* $= -35$	***[Note 7]***	**5 marks**

Notes

In the Primary method **(b)**

3 Only coordinates are acceptable for full marks. Simply stating the values at which it cuts the x- and y-axes may be awarded 1 mark (out of 2).

4 If all the coordinates are "round the wrong way" award 1 mark.

5 If the brackets are missing, treat as bad form.

In the Primary method (c)

6 Ignore any attempt to evaluate function at $x = 7/3$.

7 **•11** and **•12** are not available unless both end points and the st. points have been considered.

In the Alt.5 method (c)

8 **•12** is not available unless both end points have been considered.

In (c)

9 Some candidates simply draw up a table using integer values from –2 to 2 and make conclusions from it. This earns **•9** (Primary) ONLY, provided that one of the end points is correct.

Alternative method 5 (marks 8-12) (nature table)

•8	$6x^2 - 14x = 0$	
•9	$x = 0$ *or* $x = \frac{14}{6}$	***[Note 6]***
•10	*nature table showing* $x = 0$ *is max. tp and the greatest (maximum) value is* 9	
•11	$f(-2) = -35$ ***OR*** $f(2) = -3$	
•12	*least value* $= -35$	***[Note 8]***

9 If $\cos(2x) = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos(x)$ and $\sin(x)$. **4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
9		4	C	T8	NC	05/16

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- •1 ss use double angle formula
- •2 pd process
- •3 pd process
- •4 pd process

Primary Method : Give 1 mark for each •

- •1 $2\cos^2(x) - 1 = \frac{7}{25}$
- •2 $\cos^2(x) = \frac{32}{50}$
- •3 $\cos(x) = \frac{4}{5}$
- •4 $\sin(x) = \frac{3}{5}$

4 marks

Notes

1 In the event of $\cos^2(x) - \sin^2(x)$ being used, no marks are available until the equation reduces to a quadratic in either $\cos(x)$ *or* $\sin(x)$.

2 $\cos(x) = \pm\frac{4}{5}, \sin(x) = \pm\frac{3}{5}$ loses **•3**.

3 **•3 and •4** are only available as a consequence of attempting to apply the double angle formula. (This note does note apply to alt. method 2)

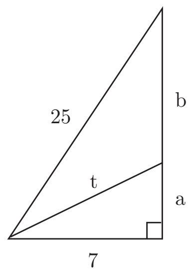
4 Guess and Check.
For guessing that $\cos(x) = \frac{4}{5}$ *and* $\sin(x) = \frac{3}{5}$, substituting them into any valid expression for $\cos(2x)$ and getting 7/25, award 1 mark only.

Alternative Method 1

- •1 $1 - 2\sin^2(x) = \frac{7}{25}$
- •2 $\sin^2(x) = \frac{18}{50}$
- •3 $\sin(x) = \frac{3}{5}$
- •4 $\cos(x) = \frac{4}{5}$

Alternative Method 2

- •1 $(7,24,25)$ triangle $\Rightarrow a + b = 24$
  and angle bisector $\Rightarrow \frac{a}{b} = \frac{7}{25}$
- •2 $a + \frac{25}{7}a = 24 \Rightarrow a = \frac{21}{4}$
- •3 $(21,28,35)$ triangle $\Rightarrow t = \frac{35}{4}$
- •4 $\cos(x) = \frac{4}{5}$ ***and*** $\sin(x) = \frac{3}{5}$



Common Error 1

$2\cos^2(x) - 1 = \frac{7}{25}$

$\cos^2(x) = \frac{64}{25}$

$\cos(x) = \frac{8}{5}$

$\sin(x) = \frac{6}{5}$

•1 ✓ •2 ×, •3 ×, •4 ×

award 1 mark only

Common Incompletion 1

- •1 ✓ $2\cos^2(x) - 1 = \frac{7}{25}$
- •2 ✓ $\cos^2(x) = \frac{32}{50}$
- •3 × $\cos(x) = \sqrt{\frac{32}{50}}$
- •4 ×✓ $\sin(x) = \sqrt{\frac{18}{50}}$

award 3 marks

10	(a)	Express $\sin(x)-\sqrt{3}\cos(x)$ in the form $k\sin(x-a)$ where $k>0$ and $0\le a\le 2\pi$.	**4**
	(b)	Hence, or otherwise, sketch the curve with equation $y=3+\sin(x)-\sqrt{3}\cos(x)$ in the interval $0\le x\le 2\pi$.	**5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10	a	4	C	T13	NC	05/27
	b	5	A	T15	NC	

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- •1 ic expand
- •2 ic compare coefficients
- •3 pd process k
- •4 pd process angle

- •5 ic state equation
- •6 ic completing graph
- •7 ic completing graph
- •8 ic completing graph
- •9 ic completing graph

Primary Method : Give 1 mark for each •

- •1 $k\sin(x)\cos(a)-k\cos(x)\sin(a)$ **STATED EXPLICITLY**
- •2 $k\cos(a)=1, k\sin(a)=\sqrt{3}$ **STATED EXPLICITLY**
- •3 $k=2$
- •4 $a=\frac{\pi}{3}$

[Notes 1-7] **4 marks**

- •5 $y=3+2\sin\left(x-\frac{\pi}{3}\right)$ **stated or implied by a correct sketch *[Note 8]***

a sketch showing ***[Notes 9,10]***

- •6 a sinusoidal curve
- •7 y-intercept at $\left(0,3-\sqrt{3}\right)$ and no x-intercepts
- •8 max at $\left(\frac{5\pi}{6},5\right)$
- •9 min at $\left(\frac{11\pi}{6},1\right)$

5 marks

Notes
In the whole question
Do not penalise more than once for not using radians.

In (a)

1 $k\left(\sin(x)\cos(a)-\cos(x)\sin(a)\right)$ is acceptable for **•1**

2 No justification is required for **•3**

3 •3 is not available for an unsimplified √4

4 $2\left(\sin(x)\cos(a)-\cos(x)\sin(a)\right)$ *or* $2\sin(x)\cos(a)-2\cos(x)\sin(a)$ is acceptabe for**•1** and **•3**

5 Candidates may use any form of the wave equation to start with as long as their final answer is in the form $k\sin(x-a)$. If it is not, then •4 is not available.

6 **•4** is only available for an answer in radians.

7 Treat $k\sin(x)\cos(a)-\cos(x)\sin(a)$ as bad form only if **•2** is gained.

In (b)

8 The **correct** sketch need not include annotation of max, min or intercept for **•5** to be awarded but you would need to see the graph lying between $y=1$ and $y=5$.

9 **•6** is available for one cycle of any sinusoidal curve of period 2π except $y=\sin(x)$. Some evidence of a scale is required.

10 For **•7,** accept 1·3 in lieu of 3 – √3

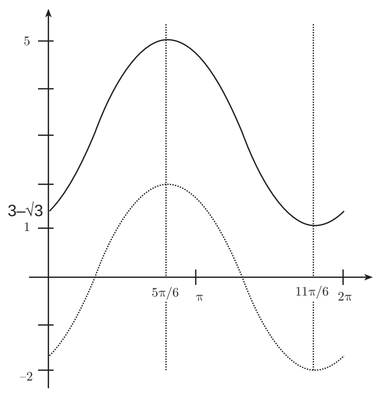
11 Do not penalise graphs which go beyond the interval 0...2π.

Alternative marking for •8 and •9

- •8 max at $x=\frac{5\pi}{6}$ ***and*** min *at* $x=\frac{11\pi}{6}$
- •9 graph lies between $y=1$ and $y=5$

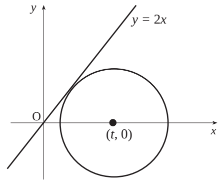
Alternative method for •5 to •9 (Calculus)

- •5 $\frac{dy}{dx}=\cos(x)+\sqrt{3}\sin(x)=0$
- •6 $\tan(x)=-\frac{1}{\sqrt{3}}$
- •7 $\max at\left(\frac{5\pi}{6},5\right)$
- •8 $\min at\left(\frac{11\pi}{6},1\right)$
- •9 $x=0\Rightarrow y=3-\sqrt{3}$ ***and*** *annotated sketch.*



11 (a) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units. Write down the equation of the circle. **1**

(b) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. **5**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11	a	1	C	G10	CN	05/28
	b	4	A	G13	CN	

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- •1 ic state equ. of circle
- •2 ss substitute
- •3 pd rearrange in standard form.
- •4 ss know to use "discriminant = 0"
- •5 ic identify "a","b" and "c"
- •6 pd process

Primary Method : Give 1 mark for each •

- •1 $(x-t)^2 + (y-0)^2 = 2^2$ **1 mark**
- •2 $(x-t)^2 + (2x)^2 = 4$
- •3 $5x^2 - 2tx + t^2 - 4 = 0$
- •4 "$b^2 - 4ac$" $= 0$ ***[Note 1]***
- •5 $a = 5,\ b = -2t,\ c = t^2 - 4$
- •6 $4t^2 - 20(t^2 - 4) = 0$

  and $t = \sqrt{5}$ ***[Note 2]*** **5 marks**

Notes

1 Subsequent to trying to use an expression masquerading as the discriminant e.g. $a^2 - 4bc = 0$, **only •5** (from the last two marks) is still available.

2 Treat $t = \pm\sqrt{5}$ as bad form.

Common Error No. 1

- •5 × $a = 5, b = -2, c = t^2 - 4$
- •6 $4 - 20(t^2 - 4) = 0$

  $20t^2 = 84$

  ×√ $t = \sqrt{\frac{21}{5}}$ *or* $\sqrt{4.2}$

Alternative Method 1 (for (b))

Let P be point of contact, C the centre of the circle. Consider triangle OPC.

- *OPC = 90° (tgt/radius)*
- *PC = 2 (radius)*
- *CP/OP = tan(COP) = 2 (gradient of tgt)*
- *Hence OP = 1*
- *and, by Pythagoras,* $t = OC = \sqrt{(2^2 + 1^2)} = \sqrt{5}$.

Alternative Method 2 (for (b))

$y = 2x \Rightarrow m_{tgt} = 2$ *and* $m_{rad} = -\frac{1}{2}$

- •2 *equ of radius is* $x + 2y = t$

  ie $x - t = -2y$
- •3 $(-2y)^2 + y^2 = 4$
- •4 $y = \frac{2}{\sqrt{5}}$
- •5 $x = \frac{1}{2}y \Rightarrow x = \frac{1}{\sqrt{5}}$
- •6 $t = x + 2y \Rightarrow t = \sqrt{5}$