

# X100/301

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NATIONAL  
QUALIFICATIONS  
2007

TUESDAY, 15 MAY  
9.00 AM – 10.10 AM

MATHEMATICS  
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

## Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



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## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

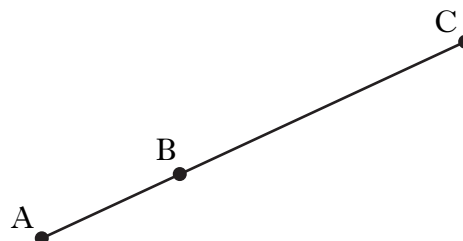
Marks

1. Find the equation of the line through the point  $(-1, 4)$  which is parallel to the line with equation  $3x - y + 2 = 0$ . 3

2. Relative to a suitable coordinate system A and B are the points  $(-2, 1, -1)$  and  $(1, 3, 2)$  respectively.

A, B and C are collinear points and C is positioned such that  $BC = 2AB$ .

Find the coordinates of C.



4

3. Functions  $f$  and  $g$ , defined on suitable domains, are given by  $f(x) = x^2 + 1$  and  $g(x) = 1 - 2x$ .

Find:

(a)  $g(f(x))$ ;

2

(b)  $g(g(x))$ .

2

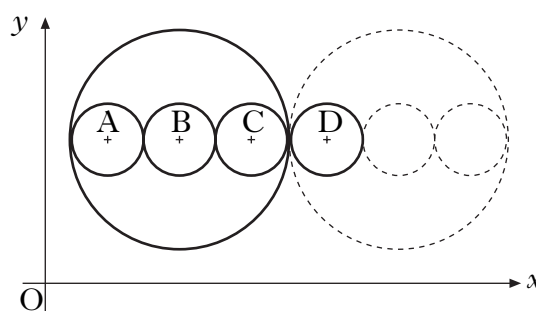
4. Find the range of values of  $k$  such that the equation  $kx^2 - x - 1 = 0$  has no real roots. 4

5. The large circle has equation  $x^2 + y^2 - 14x - 16y + 77 = 0$ .

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the  $x$ -axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



5

[Turn over

6. Solve the equation  $\sin 2x^\circ = 6\cos x^\circ$  for  $0 \leq x \leq 360$ . 4

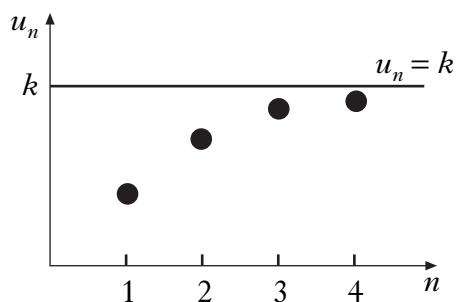
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \quad u_0 = 0.$$

(a) Calculate the values of  $u_1, u_2$  and  $u_3$ . 3

Four terms of this sequence,  $u_1, u_2, u_3$  and  $u_4$  are plotted as shown in the graph.

As  $n \rightarrow \infty$ , the points on the graph approach the line  $u_n = k$ , where  $k$  is the limit of this sequence.



(b) (i) Give a reason why this sequence has a limit.

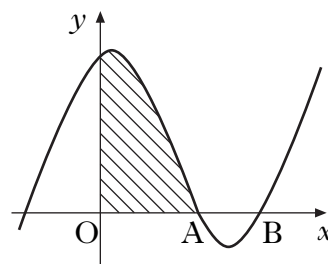
(ii) Find the exact value of  $k$ . 3

8. The diagram shows a sketch of the graph of  $y = x^3 - 4x^2 + x + 6$ .

(a) Show that the graph cuts the  $x$ -axis at  $(3, 0)$ . 1

(b) Hence or otherwise find the coordinates of A. 3

(c) Find the shaded area. 5



9. A function  $f$  is defined by the formula  $f(x) = 3x - x^3$ .  
 (a) Find the exact values where the graph of  $y = f(x)$  meets the  $x$ - and  $y$ -axes. 2

(b) Find the coordinates of the stationary points of the function and determine their nature. 7

(c) Sketch the graph of  $y = f(x)$ . 1

10. Given that  $y = \sqrt{3x^2 + 2}$ , find  $\frac{dy}{dx}$ . 3
11. (a) Express  $f(x) = \sqrt{3} \cos x + \sin x$  in the form  $k \cos(x - a)$ , where  $k > 0$  and  $0 < a < \frac{\pi}{2}$ . 4
- (b) Hence or otherwise sketch the graph of  $y = f(x)$  in the interval  $0 \leq x \leq 2\pi$ . 4

[END OF QUESTION PAPER]