2011 Mathematics

Higher

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2011 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which you will commonly see throughout your marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general you should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2 Award one mark for each •. There are no half marks.

3 Working subsequent to an error must be **followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction of mark(s) should be made.

4 **Marking Symbols**
No comments, words or acronyms should be written on scripts. Please use the following and **nothing else**.

✓ A tick should be used where a piece of working is correct and gains a mark. You are not expected to tick every line of working but you must check through the whole of a response.

___ x Where a mark is lost, the error should be underlined in **red** at the point where it first occurs, and not at any subsequent stage of the working.

✓̸ A cross-tick should be used to indicate "correct" working where a mark is awarded as a result of **follow through** from an error.

✓̸̸ A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been **eased**.

~~~ A tilde should be used to indicate a minor transgression which is not being penalised, e.g. **bad form**.

✓ with ~~~ This should be used where a candidate is given the **benefit of the doubt**.

^ A roof should be used to show that something is missing, such as a crucial step in the working or part of a solution.

These will help you to maintain consistency in your marking and will assist the examiners in the later stages of SQA procedures.

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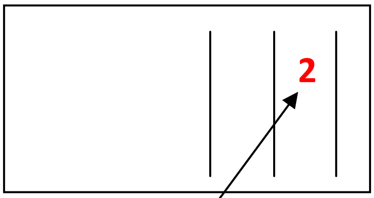
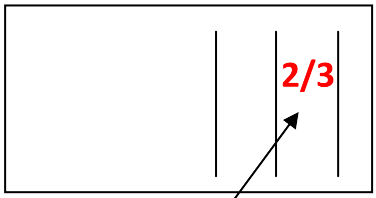
5 Regularly Occurring Responses (ROR) are shown on the marking scheme to help mark common solutions that are non-routine.

6 RORs may also be used as a guide in marking other non-routine candidate responses.

7 The mark for **each part** of a question should be entered in **red** in the **outer** right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, **as a single number**, should be written.



Marks in this column - single numbers only



Do not record marks on scripts in this manner.

8 Where a candidate has scored zero for any question, or part of a question, 0 should be written in the right hand margin beside their answer.

9 Every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol, should have a tick placed in the bottom right hand margin.

10 Where a solution is spread over several pages the marks should be recorded at the end of the solution. This should be indicated with a down arrow (↓), in the margin, at the earlier stages.

The examples below illustrate the use of the marking symbols .

Example 1

$y = x^3 - 6x^2$	$\bullet^1$	✓
$\frac{dy}{dx} = 3x^2 - 12$ ✓ $\bullet^1$ ✗ $\bullet^2$	$\bullet^2$	✗
$3x^2 - 12 = 0$ ✗̸ $\bullet^3$	$\bullet^3$	✗̸
$x = 2$ ^ $\bullet^4$	$\bullet^4$	^
$y = -16$ ✗̸ $\bullet^5$	$\bullet^5$	✗̸

Example 2

A(4,4,0), B(2,2,6), C(2,2,0)

$\overrightarrow{AB} = \mathbf{b} + \mathbf{a} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ ✗ $\bullet^1$

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix}$ ✗̸ $\bullet^2$ (repeated error)

Example 3

$3\sin x - 5\cos x$

$k\sin x\cos a - \cos x\sin a$ ✓ $\bullet^1$

$k\cos a = 3,\ k\sin a = 5$ ✓ $\bullet^2$

Example 4

Find intersection of $x + 3y = 23$ and $y = 3x - 9$

$y - 3x = 9$

$3y + x = 24$ ✓ $\bullet^1$ Strategy mark awarded. (despite two errors)

$3y - 9x = 27$

$x = -\frac{3}{10}$ ✗̸ $\bullet^2$ The subsequent pd mark is lost (Note 12)

11 Where a transcription error (paper to script or within script) occurs, a mark is lost.
e.g.

> This is a transcription error and so mark is lost.
>
> Eased as not solution of a quadratic equation.

$x^2 + 5x + 7 = 9x + 4$ ✓

$x - 4x + 3 = 0$ ✗

$x = 1$ ✓̸

> Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$x^2 + 5x + 7 = 9x + 4$ ✓

$x - 4x + 3 = 0$ ✓

$(x-3)(x-1) = 0$

$x = 1 \text{ or } 3$ ✓

12 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6 \times 6 = 12$, candidates lose the opportunity of gaining the appropriate *ic* or *pd* mark.

13 A processing error made at a strategy mark stage is penalised at the next *pd* or *ic* mark available within that part of the question. The strategy mark may still be awarded.

14 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking scheme, a correct answer with no working receives no credit.

15 Unless specifically mentioned in the marking scheme, do not penalise:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form.

16 No piece of working should be ignored without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme. Answers which are widely off-beam are unlikely to include anything of relevance, but candidates may still have the opportunity of gaining the odd mark or two, provided it satisfies the criteria for the marks.

17 In the exceptional circumstance where you are in doubt whether a mark should or should not be awarded, err on the generous side and award the mark.

18 Scored out or erased working which **has not been replaced** should be marked where still legible. However, if the scored out or erased working **has been replaced**, only the work which has not been scored out should be marked.

19 A valid approach, within Mathematical problem solving, is to try different strategies. Where this occurs, all working should be marked. The mark awarded to the candidate is from the *highest* scoring strategy. This is distinctly different from the candidate who gives two or more solutions to a question/part of a question, deliberately leaving all solutions, hoping to gain some benefit. All such contradictory responses should be marked and the *lowest* mark given.

20 It is of great importance that the utmost care should be exercised in adding up the marks.

The recommended procedure is as follows:

Step 1 Manually calculate the total from the candidate’s script.

Step 2 Check this total using the grid issued with these marking instructions.

Step 3 Input the scores and obtain confirmation of your total from the EMC screen. (This should highlight any discrepancies hitherto undiscovered.)

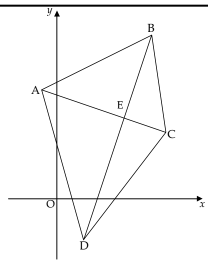
21 Place the candidate’s script for Paper 2 inside the script for Paper 1 and write the candidate’s total score (i.e. Paper 1 Section B + Paper 2) in the space provided on the front cover of the script for Paper 1.

22 In cases of difficulty, covered neither in detail nor in principle in these instructions, contact your Team Leader (TL) in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with your TL. Please see the General Marking Instructions for PA Referrals.

Question	Answer
1	C
2	B
3	D
4	D
5	A
6	C
7	D
8	A
9	B
10	D
11	D
12	C
13	C
14	B
15	B
16	A
17	A
18	C
19	C
20	D

Summary		
	A	4
	B	4
	C	6
	D	6

21 A quadrilateral has vertices A(−1, 8), B(7, 12), C(8, 5) and D(2, −3) as shown in the diagram.



(a) Find the equation of diagonal BD. **2**

(b) The equation of diagonal AC is $x + 3y = 23$.

Find the coordinates of E, the point of intersection of the diagonals. **3**

Generic Scheme			Illustrative Scheme
21 (a)			
•1	pd	find gradient of BD	•1 $\frac{15}{5}$ or equivalent
•2	ic	state equation of BD	•2 $y-(-3)=3(x-2)$ **or** $y-12=3(x-7)$
			2

Notes

1. There is no need to simplify m_{BD} for •1; however, it must be simplified before •2 can be awarded.
2. If m_{BD} cannot be simplified, due to an error, then •2 is still available.
3. Candidates who determine the equation of AC lose •1 but may still gain •2.
4. Candidates lose •1 and •2 for the equation of any side of the quadrilateral.

Regularly occurring responses

Response 1

Using $y = mx + c$

$y = 3x + c$ ✓ •1

$12 = 3\times7 + c$ **or** $-3 = 3\times2 + c$

$c = -9$ ✓ •2

2 marks out of 2

Response 2

$m_{AC} = -\frac{1}{3}$, $m_{BD} = 3$ ✗ •1

Candidate has assumed diagonals are perpendicular - without evidence.

$y-(-3)=3(x-2)$ ✗ •2

1 mark out of 2

Generic Scheme			Illustrative Scheme
21 (b)			
•3	ss	start solution of simultaneous equations	•3 e.g. $3x - y = 9$ and $x + 3y = 23$ **or** $3x - 9 = -\frac{x}{3} + \frac{23}{3}$ **or** $x + 3(3x-9) = 23$
•4	pd	solve for one variable	•4 $x = 5$ **or** $y = 6$
•5	pd	solve for second variable	•5 $y = 6$ **or** $x = 5$
			3

Notes

5. Candidates who find the equation of AC in (a), correctly or incorrectly, lose •3, •4 and •5 in (b).
6. Any other incorrect answer from (a) may still gain •3, •4 and •5 as follow through.

Regularly occurring responses

Response 3

$3x - y = 3$ and $x + 3y = 23$ ✗ •3

Subsequent to gaining •3 an error was made in simplifying the equation in (a), but strategy mark still awarded in (b).

$x = 3\cdot2$ ✗ •4

$y = 6\cdot6$ ✗ •5

Error going from (a) to (b) is penalised at first pd (or ic) mark.

2 marks out of 3

21 (c) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E. **5**

Generic Scheme			Illustrative Scheme
21 (c)			
●[6]	ss	know and find midpoint of AB	●[6] $(3,10)$
●[7]	pd	find gradient of AB	●[7] $\frac{4}{8}$ or equivalent
●[8]	ic	interpret perpendicular gradient	●[8] $-\frac{8}{4}$ or equivalent **stated, or implied by** ●[9]
●[9]	ic	state equation of perp. bisector	●[9] $y-10=-2(x-3)$ **but not** $y-6=-2(x-5)$
●[10]	ic	justification of point on line	●[10] when $x=5,\ y=-2\times5+16=6$ **or** $2\times5+6-16=0$

5

Notes

7. Candidates who do not simplify the gradient in (a) and (c) should only be penalised once.
8. ●[9] is only available as a consequence of using a midpoint and perpendicular gradient.
9. Candidates who use $y-6=-2(x-5)$ at ●[9] stage, lose ●[9] and ●[10].
10. Candidates who show that the point of intersection of BD or AC **and** the perpendicular bisector is E gain ●[10].

Regularly occurring responses

Response 4

$m_{\text{PERP BISECTOR}}=-2$
$m_{\text{"ME"}}=\ldots=-2$
So perpendicular bisector goes through E ✗ ●[10]

There must be reference to the midpoint being a common point to gain this mark.

Response 5

From (i) equation of perpendicular bisector is $y=-2x+16$, using $(3,\ 10)$.

Then in (ii) using $m=-2$ and E(5, 6) leads to $y=-2x+16$. Same equation so E lies on line. ✓ ●[10]

Response 6

From (b) $E(3\cdot2,\ 6\cdot6)$
$x=3\cdot2,\ y=\ldots=9\cdot6$, so line does not pass through E. ✗ ●[10]

Comment must be consistent with E from (b).

22 A function f is defined on the set of real numbers by $f(x) = (x-2)(x^2+1)$.

(a) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis; (ii) the y-axis. **2**

(b) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

Generic Scheme			Illustrative Scheme	
22 (a)				
•1	ic	interpret x intercept	•1 $(2, 0)$	(minimum response "(i) 2")
•2	ic	interpret y intercept	•2 $(0, -2)$	(minimum response "(ii) -2")

2

Notes

1. Candidates who obtain extra x-axis intercepts lose •1.
2. Candidates who obtain extra y-axis intercepts lose •2.
3. Candidates who interchange intercepts can gain at most one mark.

Generic Scheme			Illustrative Scheme
22 (b)			
•3	ic	write in differentiable form	•3 $x^3 - 2x^2 + x - 2$
•4	ss	know to and start to differentiate	•4 $3x^2 \ldots$ or $\ldots - 4x\ldots$
•5	pd	complete derivative and equate to 0	•5 $3x^2 - 4x + 1$ **and** $f'(x) = 0$ **or** $3x^2 - 4x + 1 = 0$
•6	pd	factorise derivative	•6 $(3x-1)(x-1)$
•7	pd	process for x	•7 $\frac{1}{3}$ and 1 ¦ $x = \frac{1}{3}$ and $y = -\frac{50}{27}$
•8	pd	evaluate y-coordinates	•8 $-\frac{50}{27}$ and -2 ¦ $x = 1$ and $y = -2$
•9	ic	justify nature of stationary points	•9 (see table below)
•10	ic	interpret and state conclusions	•10 (see table below)

•9

x	…	$\frac{1}{3}$	…	1	…
$f'(x)$	+	0	−	0	+

•10 max, min

Accept a valid expression in lieu of $f'(x)$.

8

This question may be marked vertically. The dotted rectangle shows what is required for •10.

Notes

4. •5 is only available if "$=0$" appears at or before •6 stage.
5. •3, •4 and •5 are the only marks available to candidates who solve $3x^2 - 4x = -1$.
6. At •9 the nature can be determined using the second derivative.
7. •9 is only available if the nature table is consistent with the candidate's derivative.
8. •10 is awarded for correct interpretation of the candidate's nature table in words.

Regularly occurring responses

Response 1A

x	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2
$\frac{dy}{dx}$	1	0	$-\frac{1}{4}$	0	5 ✓ •9
		max		min ✓ •10	

Response 1B

^	…	$\frac{1}{3}$	…	1	…
$f'(x)$	+	0	−	0	+ ✗ •9
		max		min ✓ •10	

x missing

Response 1C

x	…	$\frac{1}{3}$	…	1	…
slope	/	—	\	—	/

✗ •9

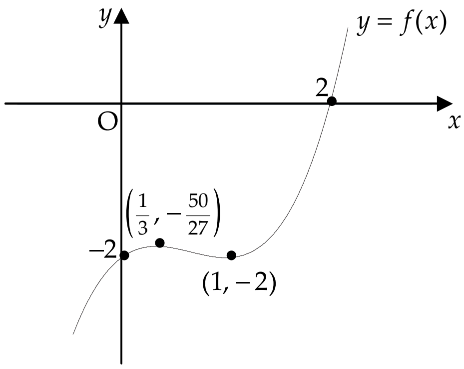
^ •10

signs or values are necessary

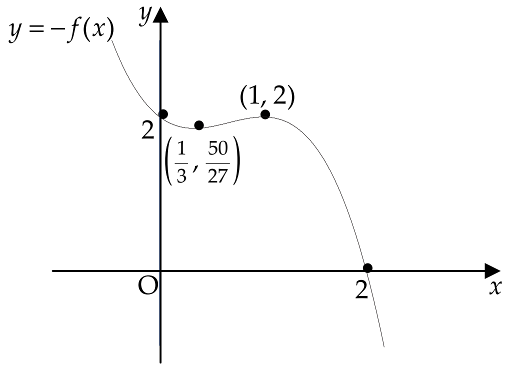
22	(c)	On separate diagrams sketch the graphs: (i) $y = f(x)$; (ii) $y = -f(x)$.	3

Generic Scheme **Illustrative Scheme**

(c) (i)



(ii)



•11	ic	curve showing points from (a) and (b) without annotation	•11 sketch
•12	ic	**cubic** curve showing **all** intercepts and stationary points annotated	•12 sketch
•13	ic	curve from (i) reflected in x − axis	•13 reflected sketch

3

Notes

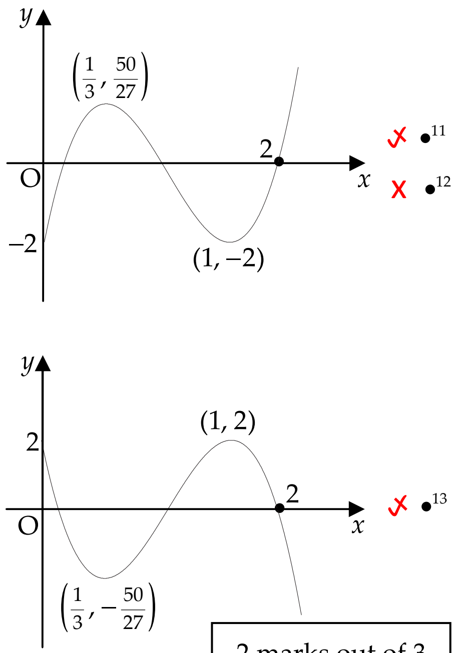
9. •11 is for any curve consistent with all points found in (a) and (b). Ignore any extra critical points.

10. In (c)(ii), the minimum requirement is the curve from (c)(i) reflected in x-axis showing **at least one** x-intercept unchanged and **at least one** stationary point correctly annotated.

Regularly occurring responses

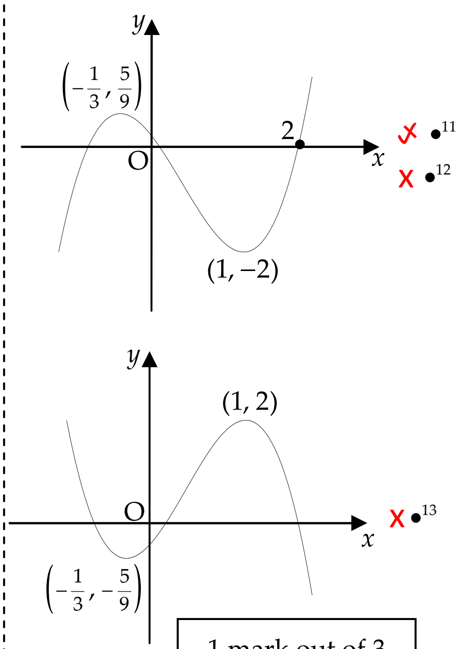
Follow through from candidate's work in (a) and (b).

Response 2



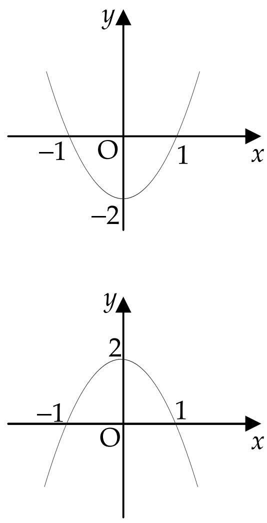
2 marks out of 3

Response 3



1 mark out of 3

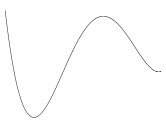
Response 4



0 marks out of 3

No marks available for a quadratic.

Response 5



✗ •12

Not a cubic due to apparent change of concavity.

23	(a)	Solve $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$ for $0 \le x < 360$.	5

Generic Scheme | **Illustrative Scheme**

23 (a)

Method 1 : Using factorisation

		Generic Scheme	Illustrative Scheme	
•1	ss	know to use double angle formula	•1 $2\cos^2 x^\circ - 1\ldots$	**stated, or implied by** •2
•2	ic	express as a quadratic in $\cos x^\circ$	•2 $2\cos^2 x^\circ - 3\cos x^\circ + 1$	= 0 must appear at either of these lines to gain •2.
•3	ss	start to solve	•3 $(2\cos x^\circ - 1)(\cos x^\circ - 1)$	

Method 2 : Using quadratic formula

•1 $2\cos^2 x^\circ - 1\ldots$

•2 $2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ **stated explicitly**

•3 $\dfrac{-(-3) \pm \sqrt{(-3)^2 - 4\times 2\times 1}}{2\times 2}$

In both methods :

		Generic Scheme	Illustrative Scheme	
•4	pd	reduce to equations in $\cos x^\circ$ only	•4 $\cos x^\circ = \frac{1}{2}$ and $\cos x^\circ = 1$	Candidates who include 360 lose •5
•5	ic	process solutions in given domain	•5 0, 60 and 300	

or

•4 $\cos x^\circ = 1$ and $x = 0$

•5 $\cos x^\circ = \frac{1}{2}$ and $x = 60$ or 300

Candidates who include 360 lose •4

5

Notes

1. •1 is not available for simply stating that $\cos 2\text{A} = 2\cos^2 \text{A} - 1$ with no further working.
2. In the event of $\cos^2 x - \sin^2 x$ or $1 - 2\sin^2 x$ being substituted for $\cos 2x$, •1 cannot be awarded until the equation reduces to a quadratic in $\cos x$.
3. Substituting $\cos 2\text{A} = 2\cos^2 \text{A} - 1$ or $\cos 2a = 2\cos^2 a - 1$ etc. should be treated as bad form throughout.
4. Candidates may express the quadratic equation obtained at the •2 stage in the form $2c^2 - 3c + 1$ or $2x^2 - 3x + 1$ etc. For candidates who do not solve a trigonometric quadratic equation at •5, $\cos x$ must appear explicitly to gain •4.
5. •4 and •5 are only available as a consequence of solving a quadratic equation.
6. Any attempt to solve $ax^2 + bx = c$ loses •3, •4 and •5.
7. •5 is not available to candidates who work in radian measure and do not convert their answers into degree measure.

Regularly occurring responses

Response 1

(Reading $\cos 2x^\circ$ as $\cos^2 x^\circ$)

$\cos^2 x^\circ - 3\cos x^\circ + 2 = 0$ ✗ •1 ✗ •2

$(\cos x^\circ - 2)(\cos x^\circ - 1) = 0$ ✓ •3

$\cos x^\circ = 2$ or $\cos x^\circ = 1$ ✓ •4

no solution $x = 0$ ✓ •5

2 marks out of 5

Response 2A

(See note 6 above)

$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$ ✓ •1

$2\cos^2 x^\circ - 3\cos x^\circ = -1$ ✗ •2

$\cos x^\circ(2\cos x^\circ - 3) = -1$ ✗ •3

$\cos x^\circ = -1$ or $\cos x^\circ = 1$ ✗ •4

$x = 180$ $x = 0$ ✗ •5

1 mark out of 5

Response 2B

(See note 6 above)

$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$ ✓ •1

$2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ ✓ •2

$2\cos^2 x^\circ - 3\cos x^\circ = -1$

$\cos x(2\cos x^\circ - 3) = -1$ ✗ •3

$\cos x^\circ = -1$ or $\cos x^\circ = 1$ ✗ •4

$x = 180$ $x = 0$ ✗ •5

2 marks out of 5

23	(b)	Hence solve $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$ for $0 \leq x < 360$.	2

Generic Scheme			Illustrative Scheme
23 (b)			
•6	ic	interpret relationship with (a)	•6 $2x = 0$ and 60 and 300
•7	ic	interpret periodicity	•7 0, 30, 150, 180, 210 and 330

2

Notes

8. Do not penalise the inclusion of 360 in (b).
9. Ignore extra answers, correct or incorrect, outside the given interval, but penalise incorrect answers within the interval.
10. Do not penalise candidates who use radians in (b) if they have already been penalised in (a).
11. Candidates who go back to 'first principles' for (b) can only gain •6 and •7 for a correct method leading to valid solutions as stated in the Illustrative Scheme.

Regularly occurring responses

Response 3A

From (a) $x = 0,\ 60,\ 300$

(b) $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$

$2(\cos 2x^\circ - 3\cos x^\circ + 1) = 0$ ✗ •6

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓̸ •7

1 mark out of 2

Response 3B

From (a) $x = 0,\ 60,\ 300$

(b) ^ •6

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓̸ •7

1 mark out of 2

Response 4A

From (a) $x = 0,\ 60,\ 300$

(b) $x \div 2 = 0,\ 30,\ 150$ ^ •6 ✗ •7

0 marks out of 2

Response 4B

From (a) $x = 0,\ 60,\ 300$

(b) $x \div 2 = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ^ •6 ✓̸ •7

1 mark out of 2

Response 5

From (a) $x = 0,\ 60,\ 300$

(b) $\cos(2.2x^\circ) - 3\cos 2x^\circ + 2 = 0$ ✓ •6

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓ •7

2 marks out of 2

Response 6

From (a) $x = 0,\ 60,\ 300$

(b) period $\div 2$ ✓ •6

so $x = 0,\ 30,\ 150,\ 180,\ 210,\ 330,\ 360,\ 570$ ✓ •7

2 marks out of 2

Response 7

From (a) $x = 0,\ 60,\ 300$

(b) $2x$ repeats every 180 ^ •6

$x = 0,\ 60,\ 300,\ 0+180,\ 60+180$

$= 0,\ 60,\ 180,\ 240,\ 300$ ✗ •7

0 marks out of 2

Response 8 (Wrong angles from (a))

e.g. $x = 0,\ 30,\ 330$

(b) $2x = 0,\ 30,\ 330$ ✓̸ •6

$x = 0,\ 15,\ 165,\ 180,\ 195,\ 345$ ✓̸ •7

2 marks out of 2