

X100/302

NATIONAL
QUALIFICATIONS
2011

WEDNESDAY, 18 MAY
10.50 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

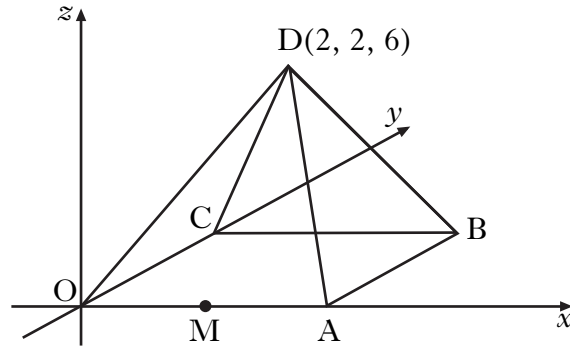
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

1. D,OABC is a square based pyramid as shown in the diagram below.



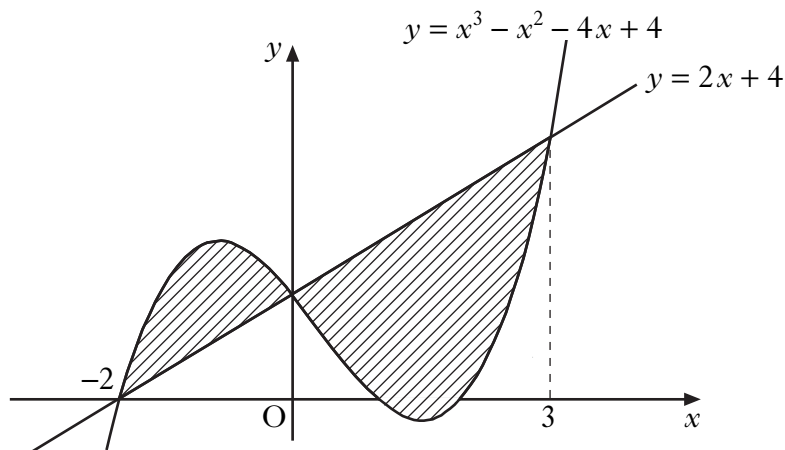
O is the origin, D is the point (2, 2, 6) and OA = 4 units.
M is the mid-point of OA.

- | | |
|----------------------------------------------------------|----------|
| (a) State the coordinates of B. | 1 |
| (b) Express \vec{DB} and \vec{DM} in component form. | 3 |
| (c) Find the size of angle BDM. | 5 |
2. Functions f , g and h are defined on the set of real numbers by
- $f(x) = x^3 - 1$
 - $g(x) = 3x + 1$
 - $h(x) = 4x - 5$.
- | | |
|---------------------------------------------------------------------|----------|
| (a) Find $g(f(x))$. | 2 |
| (b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. | 1 |
| (c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$. | |
| (ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. | 5 |
| (d) Hence solve $g(f(x)) + xh(x) = 0$. | 1 |

[Turn over

3. (a) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.
Write down the values of u_1 and u_2 . 1
- (b) A second sequence is given by 4, 5, 7, 11,
It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.
Find the values of p and q . 3
- (c) Either the sequence in (a) or the sequence in (b) has a limit.
(i) Calculate this limit.
(ii) Why does the other sequence not have a limit? 3

4. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.
The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.

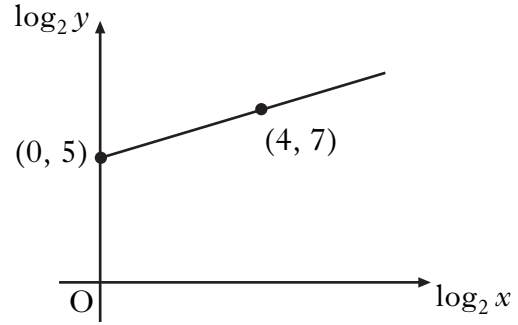


Calculate the total shaded area. 10

5. Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.

Find the values of k and n .



5

6. (a) The expression $3 \sin x - 5 \cos x$ can be written in the form $R \sin(x+a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of R and a .

4

- (b) Hence find the value of t , where $0 \leq t \leq 2$, for which

$$\int_0^t (3 \cos x + 5 \sin x) dx = 3.$$

7

7. Circle C_1 has equation $(x + 1)^2 + (y - 1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1 .

The circles have no points of contact.

What is the range of values of p ?

9

[END OF QUESTION PAPER]