

# X100/12/03

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NATIONAL  
QUALIFICATIONS 2013

WEDNESDAY, 22 MAY  
2.50 PM – 4.00 PM

MATHEMATICS  
HIGHER  
Paper 2

**Read carefully**

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

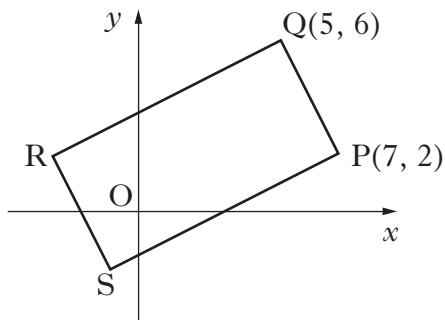
1. The first three terms of a sequence are 4, 7 and 16.  
The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of  $m$  and  $c$ .

4

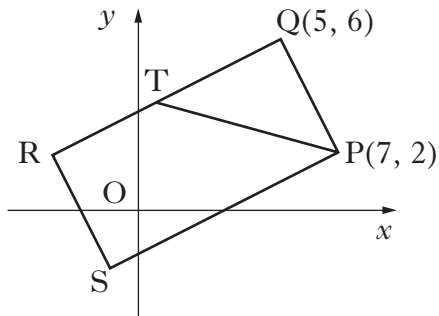
2. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



- (a) Find the equation of QR.

3

- (b) The line from P with the equation  $x + 3y = 13$  intersects QR at T.



Find the coordinates of T.

3

- (c) Given that T is the midpoint of QR, find the coordinates of R and S.

3

[Turn over

3. (a) Given that  $(x - 1)$  is a factor of  $x^3 + 3x^2 + x - 5$ , factorise this cubic fully.

4

- (b) Show that the curve with equation

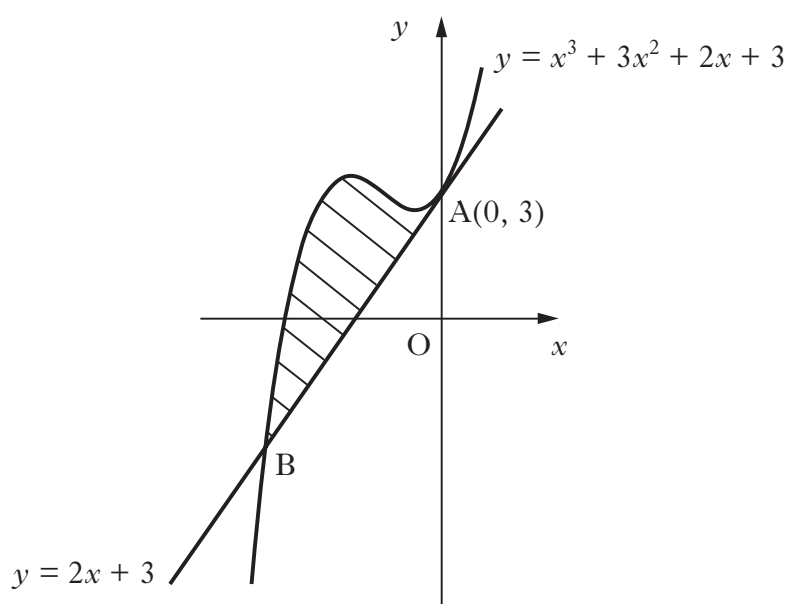
$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the  $x$ -coordinate and determine the nature of this point.

5

4. The line with equation  $y = 2x + 3$  is a tangent to the curve with equation  $y = x^3 + 3x^2 + 2x + 3$  at  $A(0, 3)$ , as shown in the diagram.



The line meets the curve again at B.

Show that B is the point  $(-3, -3)$  and find the area enclosed by the line and the curve.

6

5. Solve the equation

$$\log_5(3 - 2x) + \log_5(2 + x) = 1, \text{ where } x \text{ is a real number.}$$

4

6. Given that  $\int_0^a 5\sin 3x \, dx = \frac{10}{3}$ ,  $0 \leq a < \pi$ ,

calculate the value of  $a$ .

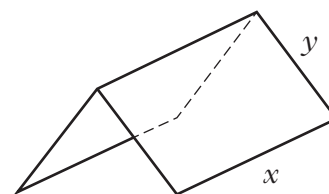
5

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- $x$  metres for top and bottom edges;
- $y$  metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is  $24 \text{ m}^2$ .

- (a) Show that the total length,  $L$  metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

3

- (b) These rods cost  $\pounds 8.25$  per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

- Find the value of  $x$  for which  $L$  is a minimum.
- Calculate the minimum cost of a frame.

7

8. Solve algebraically the equation

$$\sin 2x = 2 \cos^2 x \quad \text{for } 0 \leq x < 2\pi$$

6

[Turn over for Question 9 on Page six

9. The concentration of the pesticide,  $X_{pesto}$ , in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- $P_0$  is the initial concentration;
  - $P_t$  is the concentration at time  $t$ ;
  - $t$  is the time, in days, after the application of the pesticide.
- (a) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.  
If the half-life of  $X_{pesto}$  is 25 days, find the value of  $k$  to 2 significant figures. 4
- (b) Eighty days after the initial application, what is the percentage decrease in concentration of  $X_{pesto}$ ? 3

[END OF QUESTION PAPER]