



2014 Mathematics

Higher

Finalised Marking Instructions

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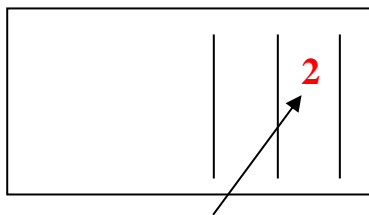
General Comments

These marking instructions are for use with the 2014 Higher Mathematics Examination.

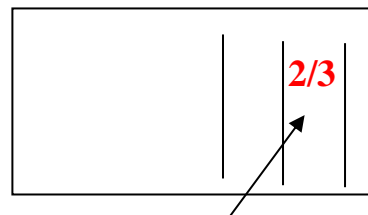
For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which are commonly seen throughout the marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general markers should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

- 1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.
- 2 Award one mark for each •. There are **no** half marks.
- 3 The mark awarded for **each part** of a question should be entered in the **outer** right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, **as a whole number**, should be written.



Marks in this column
whole numbers only



Do not record marks on
scripts in this manner.

- 4 Where a candidate has not been awarded any marks for an attempt at a question, or part of a question, 0 should be written in the right hand margin against their answer. It should not be left blank. If absolutely no attempt at a question, or part of a question, has been made, ie a completely empty space, then NR should be written in the outer margin.
- 5 IT IS ESSENTIAL that every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol should have a tick placed in the bottom right hand margin.
- 6 Where the solution to part of a question is fragmented and continues later in the script, the marks should be recorded at the end of the solution. This should be indicated with a down arrow (\downarrow), in the margin, at the earlier stages.
- 7 Working subsequent to an error **must be followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.
- 8 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

9 Marking Symbols

No comments or words should be written on scripts. Please use the following symbols and those indicated on the welcome letter and from comment 6 on the previous page.

✓ A tick should be used where a piece of working is correct and gains a mark. Markers must check through the whole of a response, ticking the work **only** where a mark is awarded.

— X At the point where an error occurs, the error should be underlined and a cross used to indicate where a mark has not been awarded. If no mark is lost the error should only be underlined, ie a cross is only used where a mark is not awarded.

✓ A cross-tick should be used to indicate “correct” working where a mark is awarded as a result of **follow through** from an error.

✗ A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been **eased**.

~~~~ A tilde should be used to indicate a minor error which is not being penalised, eg **bad form**.

✓~~~~ This should be used where a candidate is given the **benefit of the doubt**.

^ A roof should be used to show that something is missing, such as part of a solution or a crucial step in the working.

These will help markers to maintain consistency in their marking and are essential for the later stages of SQA procedures.

The examples below illustrate the use of the marking symbols .

### Example 1

$$y = x^3 - 6x^2$$

$$\frac{dy}{dx} = 3x^2 - 12 \quad \checkmark \quad \times$$

$$3x^2 - 12 = 0 \quad \checkmark \quad \times$$

$$x = 2 \quad \wedge$$

$$y = -16 \quad \checkmark \quad \times$$

- <sup>1</sup> ✓
- <sup>2</sup> ✗
- <sup>3</sup> ✓✗
- <sup>4</sup> ^
- <sup>5</sup> ✗

### Example 2

$$A(4,4,0), B(2,2,6), C(2,2,0)$$

$$\overline{AB} = \underline{\mathbf{b+a}} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \quad \times \quad \bullet^1$$

$$\overline{AC} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} \quad \checkmark \quad \bullet^2$$

### Example 3

$$3\sin x - 5\cos x$$

$$k \sin x \cos a - \cos x \sin a \quad \checkmark \quad \bullet^1$$

$$k \cos a = 3, k \sin a = 5 \quad \checkmark \quad \bullet^2$$

### Example 4

$$4 \left| \begin{array}{cccc} 1 & -5 & 2 & 8 \quad \checkmark \bullet^1 \\ & 4 & -4 & -8 \\ & 1 & 1 & -2 \quad \checkmark \bullet^2 \\ & & & 0 \quad \checkmark \bullet^2 \end{array} \right.$$

Since the remainder is 0,  $x - 4$  must be a factor. ✓<sup>3</sup>

$$(x^2 - x - 2) \quad \checkmark \bullet^4$$

$$(x - 4)(x + 1)(x - 2) \quad \checkmark \bullet^5$$

$$x = 4 \text{ or } x = -1 \text{ or } x = 2 \quad \checkmark \bullet^6$$

10 In general, as a consequence of an error perceived to be trivial, casual or insignificant, eg  $6 \times 6 = 12$ , candidates lose the opportunity of gaining a mark. But note example 4 in comment 9 and the second example in comment 11.

11 Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, eg

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation.

$$x^2 + 5x + 7 = 9x + 4 \quad \checkmark$$

$$x - 4x + 3 = 0 \quad \times$$

$$x = 1 \quad \times$$

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$$x^2 + 5x + 7 = 9x + 4 \quad \checkmark$$

$$x - 4x + 3 = 0 \quad \checkmark$$

$$(x - 3)(x - 1) = 0$$

$$x = 1 \text{ or } 3 \quad \checkmark$$

## 12 Cross marking

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme:  $\bullet^5 x = 2, x = -4$

$\bullet^6 y = 5, y = -7$

Cross marked:  $\bullet^5 x = 2, y = 5$

$\bullet^6 x = -4, y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

13 In final answers, numerical values should be simplified as far as possible.

Examples:  $\frac{15}{12}$  should be simplified to  $\frac{5}{4}$  or  $1\frac{1}{4}$        $\frac{43}{1}$  should be simplified to 43

$\frac{15}{0.3}$  should be simplified to 50

$\frac{4}{\frac{5}{3}}$  should be simplified to  $\frac{4}{15}$

$\sqrt{64}$  must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

14 Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide in marking similar non-routine candidate responses.

15 Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, eg angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form;
- Repeated error within a question, but not between questions or papers.

- 16** In any ‘Show that . . .’ question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow through from a previous error.
- 17** All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate’s response. Marks may still be available later in the question so reference must be made continually to the marking instructions. All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.
- 18** Scored out working which **has not been replaced** should be marked where still legible. However, if the scored out working **has been replaced**, only the work which has not been scored out should be marked.
- 19** Where a candidate has made multiple attempts using the same strategy, mark all attempts and award the lowest mark.  
Where a candidate has tried different strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark. For example:

|                                                                    |                                                                    |
|--------------------------------------------------------------------|--------------------------------------------------------------------|
| Strategy 1 attempt 1 is worth 3 marks                              | Strategy 2 attempt 1 is worth 1 mark                               |
| Strategy 1 attempt 2 is worth 4 marks                              | Strategy 2 attempt 2 is worth 5 marks                              |
| From the attempts using strategy 1, the resultant mark would be 3. | From the attempts using strategy 2, the resultant mark would be 1. |

In this case, award 3 marks.

- 20** It is of great importance that the utmost care should be exercised in totalling the marks.  
A tried and tested procedure is as follows:
- Step 1     Manually calculate the total from the candidate’s script.
- Step 2     Check this total using the grid issued with these marking instructions.
- Step 3     Electronically enter the marks and obtain a total, which should now be compared to the manual total.

This procedure enables markers to identify and rectify any errors in data entry before submitting each candidate’s marks.

- 21** **The candidate’s script for Paper 2 should be placed inside the script for Paper 1, and the candidate’s total score (ie Paper 1 Section B + Paper 2) written in the space provided on the front cover of the script for Paper 1.**

**Paper 1 Section A**

| <b><u>Question</u></b> | <b><u>Answer</u></b> |
|------------------------|----------------------|
| <b>1</b>               | <b>C</b>             |
| <b>2</b>               | <b>B</b>             |
| <b>3</b>               | <b>A</b>             |
| <b>4</b>               | <b>D</b>             |
| <b>5</b>               | <b>D</b>             |
| <b>6</b>               | <b>A</b>             |
| <b>7</b>               | <b>C</b>             |
| <b>8</b>               | <b>D</b>             |
| <b>9</b>               | <b>B</b>             |
| <b>10</b>              | <b>C</b>             |
| <b>11</b>              | <b>C</b>             |
| <b>12</b>              | <b>A</b>             |
| <b>13</b>              | <b>B</b>             |
| <b>14</b>              | <b>D</b>             |
| <b>15</b>              | <b>B</b>             |
| <b>16</b>              | <b>C</b>             |
| <b>17</b>              | <b>B</b>             |
| <b>18</b>              | <b>C</b>             |
| <b>19</b>              | <b>A</b>             |
| <b>20</b>              | <b>D</b>             |
| <br>                   |                      |
| <b><u>Summary</u></b>  |                      |
| <b>A</b>               | <b>4</b>             |
| <b>B</b>               | <b>5</b>             |
| <b>C</b>               | <b>6</b>             |
| <b>D</b>               | <b>5</b>             |

**Paper 1- Section B**

| Question       |    | Generic Scheme                                 | Illustrative Scheme                                           | Max Mark |
|----------------|----|------------------------------------------------|---------------------------------------------------------------|----------|
| 21             | a  |                                                |                                                               |          |
| • <sup>1</sup> | ss | know to differentiate and one term correct     | • <sup>1</sup> = 6x.... or = .... - 3x <sup>2</sup>           |          |
| • <sup>2</sup> | ss | the other term correct and set derivative to 0 | • <sup>2</sup> 6x - 3x <sup>2</sup> = 0 stated explicitly     |          |
| • <sup>3</sup> | pd | solve $\frac{dy}{dx} = 0$                      | • <sup>3</sup> $x = 0$ • <sup>4</sup> 2                       |          |
| • <sup>4</sup> | pd | evaluate <b>y</b> coordinates                  | • <sup>4</sup> $y = 0$ 4                                      |          |
| • <sup>5</sup> | pd | justify nature of stationary points            | • <sup>5</sup> use 2 <sup>nd</sup> derivative or nature table |          |
| • <sup>6</sup> | ic | interpretation                                 | • <sup>6</sup> min. at (0,0) and max. at (2,4)                | <b>6</b> |

**Notes:**

- <sup>2</sup> is not available for statements such as ' $\frac{dy}{dx} = 0$ ' with no other working.
- Accept  $3x^2 - 6x = 0$  for •<sup>2</sup>.
- For candidates using a nature table, the minimum response for •<sup>5</sup> is:  
 $x$  values 0 and 2;  $\frac{dy}{dx}$  or expression  $6x - 3x^2$ ; signs and zeroes; shape.
 

|  |   |   |   |
|--|---|---|---|
|  | 0 | 2 |   |
|  | - | + | - |
|  | \ | / | \ |
- For candidates who differentiate correctly but then solve  $\frac{dy}{dx} = 0$  incorrectly, •<sup>4</sup> may be awarded as a follow through mark. •<sup>5</sup> and •<sup>6</sup> are not available if a nature table has been used, but may be awarded where candidates have used the 2<sup>nd</sup> derivative.
- For candidates who differentiate incorrectly •<sup>3</sup> and •<sup>4</sup> may be awarded as follow through marks. •<sup>5</sup> and •<sup>6</sup> are not available if a nature table has been used, but may be awarded where candidates have used the 2<sup>nd</sup> derivative.
- At •<sup>6</sup> stage accept min at  $x = 0$  and max at  $x = 2$ .
- Candidates who find the  $x$ -coordinates of the SPs correctly but correctly process only one of these to determine its nature, gain •<sup>6</sup> but not •<sup>5</sup>.

**Commonly Observed Responses:**

**Candidate A**

$$\frac{d^2y}{dx^2} = 6 - 6x$$

at  $x = 0$ ,  $\frac{d^2y}{dx^2} > 0$ , at  $x = 2$ ,  $\frac{d^2y}{dx^2} < 0$

hence minimum SP at  $x = 0$ , maximum SP at  $x = 2$

•<sup>5</sup> ✓

•<sup>6</sup> ✓

**Candidate B**

$$\frac{dy}{dx} = 6x - 3x^2 = 0 \quad \bullet^1 \quad \checkmark \quad \bullet^2 \quad \checkmark$$

$$3x(3-x) = 0$$

$$x = 0, x = 3 \quad \bullet^3 \quad \times$$

$$y = 0, y = 0 \quad \bullet^4 \quad \checkmark$$

**Case (i)**

$$\frac{d^2y}{dx^2} = 6 - 6x$$

$$x = 0 \Rightarrow \frac{d^2y}{dx^2} > 0 \Rightarrow \text{Minimum SP} \quad \bullet^5 \quad \checkmark$$

$$x = 3 \Rightarrow \frac{d^2y}{dx^2} < 0 \Rightarrow \text{Maximum SP} \quad \bullet^6 \quad \checkmark$$

**Case (ii)**

$$x \mid \rightarrow 0 \rightarrow 3 \rightarrow$$

$$\frac{dy}{dx} \mid - 0 \quad ? \quad ? \quad +$$

•<sup>5</sup> × •<sup>6</sup> ×

? inconsistent. Different signs for  $6x - 3x^2$  or  $3x(3-x)$

**21**

**b**

•<sup>7</sup> pd find intercepts

•<sup>8</sup> ic sketch

•<sup>7</sup>  $3x^2 - x^3 = 0$  **and** (3,0) or  $x = 3$ ;  
(0,0) [may appear in part a]

•<sup>8</sup> sketch

**2**

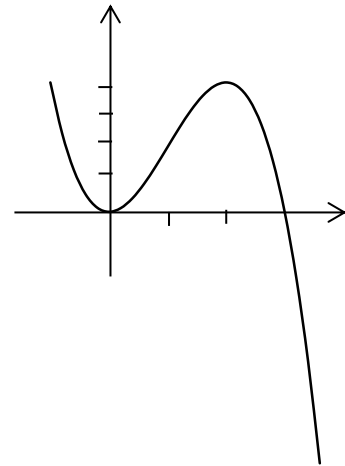
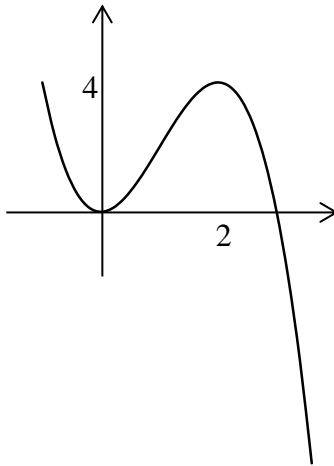
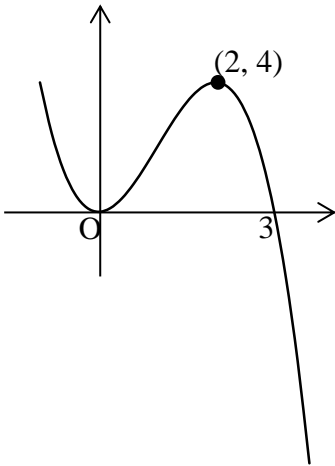
**Notes:**

8. •<sup>7</sup> accept  $3x^2 - x^3 = 0$  **and** correctly annotated diagram with 0, 3 and no other intercepts marked on sketch.

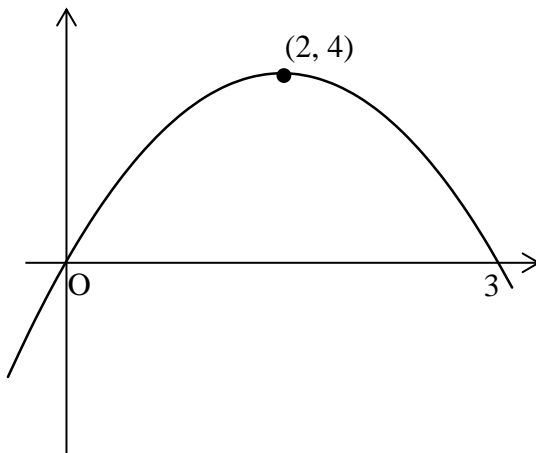
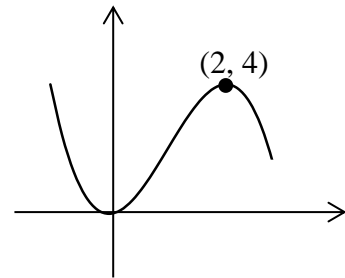
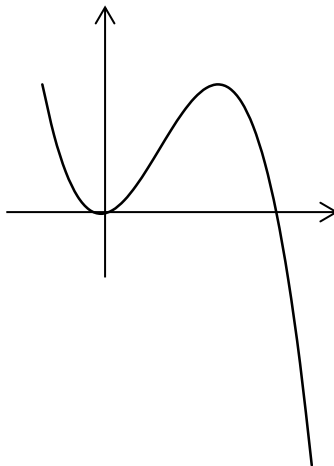
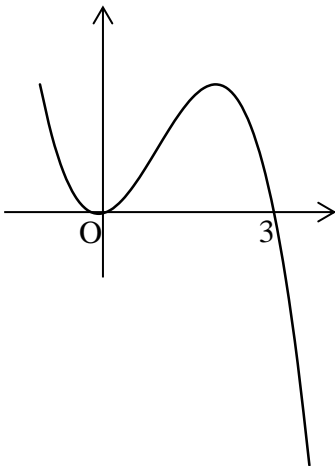
9. The minimum required for •<sup>8</sup> is a cubic curve, consistent with the SPs found in part (a) and appropriate number of  $x$  intercepts appearing on their sketch. It must be possible to determine the coordinates of the SPs from the sketch.

**Commonly Observed Responses:**

The following are acceptable for  $\bullet^8$



**Do not accept the following for  $\bullet^8$**



| Question |   | Generic Scheme                                                                                                                                                                                                                                                                                                         | Illustrative Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Max Mark |
|----------|---|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| 22       | a |                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |          |
|          |   | <ul style="list-style-type: none"> <li>•<sup>1</sup> ss know to use <math>x = -1</math> and obtain an equation</li> <li>•<sup>2</sup> ss know to use <math>x = 2</math> and obtain an equation</li> <li>•<sup>3</sup> pd process equations to find one value</li> <li>•<sup>4</sup> pd find the other value</li> </ul> | <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>6(-1)^3 + 7(-1)^2 + a(-1) + b = 0</math></li> <li>•<sup>2</sup> <math>6(2)^3 + 7(2)^2 + a(2) + b = 72</math></li> <li>•<sup>3</sup> <math>a = -1</math> or <math>b = -2</math></li> <li>•<sup>4</sup> <math>b = -2</math> or <math>a = -1</math></li> </ul> <p style="text-align: center;"><b>Alternative Method for •<sup>1</sup> and •<sup>2</sup></b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math display="block">  \begin{array}{r rrrr}  -1 &amp; 6 &amp; 7 &amp; a &amp; b \\  &amp; &amp; -6 &amp; -1 &amp; -a+1 \\  \hline  &amp; 6 &amp; 1 &amp; a-1 &amp; b-a+1=0  \end{array}  </math> </li> <li>•<sup>2</sup> <math display="block">  \begin{array}{r rrrr}  2 &amp; 6 &amp; 7 &amp; a &amp; b \\  &amp; &amp; 12 &amp; 38 &amp; 2a+76 \\  \hline  &amp; 6 &amp; 19 &amp; a+38 &amp; 2a+b+76=72  \end{array}  </math> </li> </ul> | <b>4</b> |

**Note:**

1. An incorrect value at •<sup>3</sup> should be followed through for the possible award of •<sup>4</sup>. However, if the equations are such that no solution exists, then •<sup>3</sup> and •<sup>4</sup> are not available.

**Commonly Observed Responses:**

**Candidate A**

•<sup>1</sup> ✗

$$\begin{array}{r|rrrr}
 1 & 6 & 7 & a & b \\
 & & 6 & 13 & a+13 \\
 \hline
 & 6 & 13 & a+13 & a+b+13=0
 \end{array}$$

•<sup>2</sup> ✗ repeated error

$$\begin{array}{r|rrrr}
 -2 & 6 & 7 & a & b \\
 & & -12 & 10 & -2a-20 \\
 \hline
 & 6 & -5 & a+10 & -2a+b-20=72
 \end{array}$$

Solving to get  $a = -35, b = 22$       •<sup>3</sup> ✗      •<sup>4</sup> ✗

Leading to, in part (b),  $\Rightarrow 6x^3 + 7x^2 - 35x + 22 = (x-1)(6x^2 + 13x - 22)$

•<sup>5</sup> ✗      •<sup>6</sup> ✗      •<sup>7</sup> ^

| Question       |    | Generic Scheme                                         | Illustrative Scheme                                                                      | Max Mark |
|----------------|----|--------------------------------------------------------|------------------------------------------------------------------------------------------|----------|
| 22             | b  |                                                        |                                                                                          |          |
| • <sup>5</sup> | ss | substitute for $a$ and $b$ and know to divide by $x+1$ | • <sup>5</sup> $(6x^3 + 7x^2 - x - 2) \div (x+1)$<br>Stated or implied by • <sup>6</sup> | 3        |
| • <sup>6</sup> | pd | obtain quadratic factor                                | • <sup>6</sup> $(x+1)(6x^2 + x - 2)$                                                     |          |
| • <sup>7</sup> | pd | complete factorisation                                 | • <sup>7</sup> $(x+1)(3x+2)(2x-1)$                                                       |          |

### Notes:

- For candidates who substitute  $a = -1$  into the correct quotient from part (a), •<sup>5</sup>, •<sup>6</sup> and •<sup>7</sup> are available.
- Candidates who use incorrect values obtained in part (a) may gain •<sup>5</sup>, •<sup>6</sup> and •<sup>7</sup>
- Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that  $b^2 - 4ac < 0$  to gain •<sup>7</sup>.
- Do not penalise the inclusion of ' $= 0$ ' or for solving for  $x$ .
- Candidates who use values, ex nihilo, for  $a$  and  $b$  can gain •<sup>5</sup>, if division is correct, but •<sup>6</sup> and •<sup>7</sup> are only available if  $(x+1)$  is a factor of the resulting expression.

### Commonly Observed Responses:

#### Candidate B

22a no solution

22b  $a = -4, b = -5$  ex nihilo

$$(6x^3 + 7x^2 - 4x - 5) \div (x+1)$$

|    |   |    |    |    |
|----|---|----|----|----|
| -1 | 6 | 7  | -4 | -5 |
|    |   | -6 | -1 | 5  |
|    | 6 | 1  | -5 | 0  |

$(x+1)(6x^2 + x - 5)$

$(x+1)(6x-5)(x+1)$

•<sup>5</sup> ✗

•<sup>6</sup> ✗

•<sup>7</sup> ✗

#### Candidate C

22a no solution

22b  $a = 2, b = 3$  ex nihilo

$$(6x^3 + 7x^2 + 2x + 3) \div (x+1)$$

|    |   |    |    |    |
|----|---|----|----|----|
| -1 | 6 | 7  | 2  | 3  |
|    |   | -6 | -1 | -1 |
|    | 6 | 1  | 1  | 2  |

$\Rightarrow (x+1)$  is not a factor

•<sup>5</sup> and •<sup>7</sup> are not available

•<sup>5</sup> ✗

#### Candidate D

22a no solution

22b  $a = 4, b = 3$  ex nihilo

$$(6x^3 + 7x^2 + 4x + 3) \div (x+1)$$

|    |   |    |    |    |
|----|---|----|----|----|
| -1 | 6 | 7  | 4  | 3  |
|    |   | -6 | -1 | -3 |
|    | 6 | 1  | 3  | 0  |

$(x+1)(6x^2 + x + 3)$

$b^2 - 4ac = 1 - 72 = -71$

$-71 < 0$  so does not factorise

•<sup>5</sup> ✗

•<sup>6</sup> ✗

•<sup>7</sup> ✗

| Question                                                                                                                                                                                                                                                                           | Generic Scheme                                                                                                                                                                                                                                                                                                    | Illustrative Scheme | Max Mark |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|----------|
| 23 a                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                   |                     |          |
| <ul style="list-style-type: none"> <li>•<sup>1</sup> ss substitute <math>3x - 5</math></li> <li>•<sup>2</sup> pd express in standard quadratic form</li> <li>•<sup>3</sup> pd find <math>x</math>-coordinates</li> <li>•<sup>4</sup> pd find <math>y</math>-coordinates</li> </ul> | <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^2 + (3x - 5)^2 + 2x - 4(3x - 5) - 15 = 0</math></li> <li>•<sup>2</sup> <math>10x^2 - 40x + 30 = 0</math></li> <li>•<sup>3</sup> <math>x = 1</math>    <math>x = 3</math></li> <li>•<sup>4</sup> <math>y = -2</math>    <math>y = 4</math></li> </ul> | 4                   |          |

**Notes:**

- '= 0' must appear at •<sup>1</sup> or •<sup>2</sup> for mark •<sup>2</sup> to be awarded.
- If  $x = \frac{1}{3}(y + 5)$  is substituted at •<sup>1</sup> then  $10y^2 - 20y - 80 = 0$  is obtained at •<sup>2</sup>.
- Special Case:** In cases where  $x = 1$  and  $x = 3$  do not appear as a result of •<sup>1</sup> and •<sup>2</sup>, but are substituted into the equation of the line to obtain the  $y$  values, **if** the candidate then checks that both points lie on the circle,  $\frac{3}{4}$  marks are awarded. If, in addition, the candidate makes a statement to the effect that a line can only cut a circle in, at most, 2 points, then  $\frac{4}{4}$  marks are awarded. Otherwise,  $\frac{0}{4}$  marks.
- <sup>3</sup> and •<sup>4</sup> are not available for any attempt to solve a quadratic equation of the form  $ax^2 + bx = c$

**Commonly Observed Responses:**

**Candidate A**

|                                              |                                       |  |
|----------------------------------------------|---------------------------------------|--|
| $x^2 + (3x - 5)^2 + 2x - 4(3x - 5) - 15 = 0$ | • <sup>1</sup> ✓                      |  |
| $10x^2 - 40x + 40 = 0$                       | • <sup>2</sup> ✗                      |  |
| $x = 2$ and $y = 1$                          | • <sup>3</sup> ✓✓    • <sup>4</sup> ^ |  |

|                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |   |  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|--|
| 23 b                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |   |  |
| <ul style="list-style-type: none"> <li>•<sup>5</sup> ss state centre</li> <li>•<sup>6</sup> pd calculate gradients</li> <li>•<sup>7</sup> ic communicate result</li> </ul> | <ul style="list-style-type: none"> <li>•<sup>5</sup> <math>(-1, 2)</math></li> <li>•<sup>6</sup> <math>m = -2, m = \frac{1}{2}</math></li> <li>•<sup>7</sup> demonstrates<br/><math>m_1 \times m_2 = -2 \times \frac{1}{2} = -1</math><br/><math>\Rightarrow</math> PT is perpendicular to QT<br/>[or other appropriate statement]</li> </ul> <p style="text-align: center;"><b>Alternative Method</b></p> <ul style="list-style-type: none"> <li>•<sup>5</sup> <math>(-1, 2)</math></li> <li>•<sup>6</sup> eg <math>\begin{pmatrix} -2 \\ 4 \end{pmatrix}</math> and <math>\begin{pmatrix} -4 \\ -2 \end{pmatrix}</math></li> <li>•<sup>7</sup> <math>\begin{pmatrix} -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -2 \end{pmatrix} = -2 \times -4 + 4 \times -2 = 0</math><br/><math>\Rightarrow</math> PT is perpendicular to QT<br/>[or other appropriate statement]</li> </ul> | 3 |  |

**Notes:**

4. Other valid strategies:
- Converse of Pythagoras' Theorem:
    - <sup>6</sup> process lengths,  $PT = QT = \sqrt{20}$ ,  $PQ = \sqrt{40}$
    - <sup>7</sup> apply converse and communicate result clearly.
  - Cosine Rule:
    - <sup>6</sup> process lengths, •<sup>7</sup> apply cosine rule to obtain angle  $90^\circ$  and communicate result clearly.

**Commonly Observed Responses:**

| Candidate B                                             | Candidate C                                      |
|---------------------------------------------------------|--------------------------------------------------|
| T(-1,2)                      • <sup>5</sup> ✓           | T(-1,2)                      • <sup>5</sup> ✓    |
| $m = \frac{1}{2}, m = -2$ • <sup>6</sup> ✓              | $m_1 = \frac{1}{2}, m_2 = -2$ • <sup>6</sup> ✓   |
| $m_1 \times m_2 = -1$ • <sup>7</sup> ^                  | $m_1 \times m_2 = -1$ • <sup>7</sup> ✓           |
| No link between required condition and gradients found. | Required condition is linked to gradients found. |

| 23                                                                  | c                                                   |  |          |
|---------------------------------------------------------------------|-----------------------------------------------------|--|----------|
| • <sup>8</sup> ss knows to find and states centre                   | • <sup>8</sup> centre (2, 1)                        |  | <b>3</b> |
| • <sup>9</sup> pd calculate radius                                  | • <sup>9</sup> radius = $\sqrt{10}$                 |  |          |
| • <sup>10</sup> ic state equation of circle                         | • <sup>10</sup> $(x - 2)^2 + (y - 1)^2 = 10$        |  |          |
|                                                                     | <b>Alternative Method</b>                           |  |          |
| • <sup>8</sup> ss substitute points into general equation of circle | $x^2 + y^2 + 2gx + 2fy + c = 0$                     |  |          |
|                                                                     | • <sup>8</sup> $25 + 6g + 8f + c = 0$               |  |          |
|                                                                     | $5 + 2g - 4f + c = 0$                               |  |          |
|                                                                     | $5 - 2g + 4f + c = 0$                               |  |          |
| • <sup>9</sup> pd find $f$ or $g$ or $c$                            | • <sup>9</sup> $f = -1$ , or $g = -2$ , or $c = -5$ |  |          |
| • <sup>10</sup> ic state values of $f$ , $g$ and $c$                | • <sup>10</sup> $f = -1, g = -2, c = -5$            |  |          |

**Notes:**

- $(\sqrt{10})^2$  must be simplified to gain •<sup>10</sup>
- For candidates who find P and Q correctly in part (a), award •<sup>8</sup> if centre (2,1) appears without working.
- For the mid-point of PQ being (2,1), •<sup>8</sup> is available unless subsequent working indicates that this is not the intended centre.
- <sup>9</sup> is only available as a result of PQ being a diameter, or using a valid strategy to find the centre eg midpoint of PQ or point of intersection of the perpendicular bisectors of PT and TQ. •<sup>10</sup> is still available.
- Where an incorrect centre or an incorrect radius appear ex nihilo •<sup>10</sup> is not available.

| Question                                                                                                                                                                                                                                                                                                                                                                                                                              | Generic Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Illustrative Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Max Mark |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| 24                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                       | <ul style="list-style-type: none"> <li>•<sup>1</sup> ss take <math>\log_9</math> of both sides of the equation</li> <li>•<sup>2</sup> pd apply laws of logarithms</li> <li>•<sup>3</sup> pd apply laws of logarithms</li> <li>•<sup>4</sup> pd find <math>k</math></li> <li>•<sup>5</sup> pd find <math>a</math></li> <br/> <li>•<sup>1</sup> ss know to use equation of the line</li> <li>•<sup>2</sup> pd write in exponential form</li> <li>•<sup>3</sup> pd apply laws of indices</li> <li>•<sup>4</sup> pd find <math>k</math></li> <li>•<sup>5</sup> pd find <math>a</math></li> </ul> | <p style="text-align: center;"><b>Method 1</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\log_9 y = \log_9 ka^x</math></li> <li>•<sup>2</sup> <math>\log_9 y = \log_9 k + \log_9 a^x</math></li> <li>•<sup>3</sup> <math>\log_9 y = \log_9 k + x \log_9 a</math></li> <li>•<sup>4</sup> <math>\log_9 k = 2, k = 81</math> or <math>k = 9^2 = 81</math></li> <li>•<sup>5</sup> <math>\log_9 a = \frac{1}{2}, a = 3</math> or <math>a = 9^{1/2} = 3</math></li> </ul> <p style="text-align: center;"><b>Method 2</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\log_9 y = \frac{1}{2}x + 2</math></li> <li>•<sup>2</sup> <math>y = 9^{2 \times \frac{1}{2}x + 2}</math></li> <li>•<sup>3</sup> <math>y = 9^{\frac{1}{2}x} 9^2</math></li> <li>•<sup>4</sup> <math>k = 81</math></li> <li>•<sup>5</sup> <math>a = 3</math></li> </ul> | <b>5</b> |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |          |
| <ol style="list-style-type: none"> <li>1. Candidates who start with •<sup>3</sup> <math>\log_9 y = \log_9 k + x \log_9 a</math> also gain •<sup>1</sup> and •<sup>2</sup>.</li> <li>2. In Method 1, base 9 must appear by •<sup>4</sup> stage, for •<sup>1</sup> to be awarded.</li> <li>3. For <math>k = 81</math> and <math>a = 3</math> with spurious or no working, •<sup>4</sup> and •<sup>5</sup> are not available.</li> </ol> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |          |

## Commonly Observed Responses:

### Candidate A

|                              |               |                                                                               |
|------------------------------|---------------|-------------------------------------------------------------------------------|
| $\log y = \log ka^x$         | $\bullet^1$ ✗ | See Note 2                                                                    |
| $\log y = \log k + \log a^x$ | $\bullet^2$ ✓ |                                                                               |
| $\log y = \log k + x \log a$ | $\bullet^3$ ✓ |                                                                               |
| $k = 81$                     | $\bullet^4$ ✗ | No evidence of which base is being used.                                      |
| $a = 3$                      | $\bullet^5$ ✓ | Answers at both $\bullet^4$ and $\bullet^5$ are consistent with using base 9. |

### Candidate B : A combination of Method 1 and Method 2.

|                                   |                                                |  |               |               |
|-----------------------------------|------------------------------------------------|--|---------------|---------------|
| M2                                | $\log_9 y = \frac{1}{2}x + 2$                  |  | $\bullet^1$ ✓ |               |
| M1                                | $\log_9 y = \log_9 ka^x$                       |  | $\bullet^2$ ✓ | $\bullet^3$ ✓ |
|                                   | $\Rightarrow \log_9 y = x \log_9 a + \log_9 k$ |  |               |               |
| equating gradients and intercepts |                                                |  |               |               |
|                                   | $\log_9 a = \frac{1}{2}$                       |  |               |               |
|                                   | $a = 9^{\frac{1}{2}} = 3$                      |  | $\bullet^4$ ✓ |               |
|                                   | $\log_9 k = 2$                                 |  | $\bullet^5$ ✓ |               |
|                                   | $k = 9^2 = 81$                                 |  |               |               |

### Candidate C

|                            |               |                               |               |
|----------------------------|---------------|-------------------------------|---------------|
| at (0,2) $\log_9 y = 2$    |               | at (6,5) $\log_9 y = 5$       |               |
| $\Rightarrow y = 9^2 = 81$ | $\bullet^1$ ✓ | $\Rightarrow y = 9^5$         | $\bullet^3$ ✓ |
| Substitute into equation   |               | substitute into equation      |               |
| $81 = ka^0$                |               | $9^5 = ka^6$                  |               |
| $\Rightarrow k = 81$       | $\bullet^2$ ✓ | $\Rightarrow 9^5 = 81a^6$     | $\bullet^4$ ✓ |
|                            |               | $\Rightarrow a^6 = 9^3 = 3^6$ |               |
|                            |               | $\Rightarrow a = 3$           | $\bullet^5$ ✓ |