Paper 2

Question		Generic Scheme			Illustrative Scheme	Max Mark
1	**a**					
	$\bullet^1$	ss	find gradient of AB	$\bullet^1$	$m_{AB}=1$	
	$\bullet^2$	pd	find perpendicular gradient	$\bullet^2$	$m_{perp}=-1$ stated or implied by $\bullet^4$	
	$\bullet^3$	pd	find midpoint of AB	$\bullet^3$	$(4,1)$ stated or implied by $\bullet^4$	
	$\bullet^4$	pd	obtain equation	$\bullet^4$	$y-1=-1(x-4)$	**4**

Notes:

1. $\bullet^4$ is only available as a consequence of using a perpendicular gradient **and** a midpoint.
2. The gradient must appear in simplified form at $\bullet^4$ stage for $\bullet^4$ to be awarded.

Commonly Observed Responses:

Candidate A

$m_{AB}=-1$ $\bullet^1$ ✗

$m_{perp}=1$ $\bullet^2$ $\cancel{\checkmark}$

$(4,1)$ $\bullet^3$ ✓

$y-1=1(x-4)\ \Rightarrow y=x-3$ $\bullet^4$ $\cancel{\checkmark}$

Leading to part (b)

$y-x=-3$
$y+2x=6$ $\bullet^5$ $\cancel{\checkmark}$

$(3,0)$ $\bullet^6$ $\cancel{\checkmark}$

$\bullet^7$ and $\bullet^8$ are not available as $\mathrm{A}=\mathrm{T}=(3,0)$

Question		**Generic Scheme**		**Illustrative Scheme**	**Max Mark**
1	**b**				
$\bullet^5$	ss	know to solve simultaneously	$\bullet^5$	$y+2x=6$ $y+x=5$	
$\bullet^6$	pd	solve correctly for x and y	$\bullet^6$	$x=1,\ y=4$	**2**

Commonly Observed Responses:

Candidate B

Part (a) $y-1=-1(x-4)$ $\bullet^4$ ✓

$y=-x+3$ error

Part (b) $y+2x=6$ and $y+x=3$ $\bullet^5$ ✓

$x=3,\ y=0$ $\bullet^6$ ✓̸ correct strategy used, pd mark not available

Question		**Generic Scheme**		**Illustrative Scheme**	**Max Mark**
1	**c**				
$\bullet^7$	ss	know and use $m=\tan\theta$	$\bullet^7$	$\tan\theta=-2$	
$\bullet^8$	pd	calculate angle	$\bullet^8$	$116\cdot6^{\circ}$ accept 117^0 or $2\cdot03$ radians	**2**

Commonly Observed Responses:

Candidate C

$m_{\text{AT}}=-\frac{1}{2}$

base angle $=26\cdot6^{\circ}$ $\bullet^7$ ✗

$\Rightarrow$ angle $=90+26\cdot6=116\cdot6^{\circ}$ $\bullet^8$ ✗

Candidate D

$m_{\text{AT}}=2$ $\bullet^7$ ✗

angle $=\tan^{-1}(2)=63\cdot4^{\circ}$ $\bullet^8$ ✓̸

Candidate E:

Part (a)

$m_{\text{AB}}=\frac{2-0}{5-3}=\frac{2}{8}=\frac{1}{4}$ $\bullet^1$ ✗

$m_{\text{perp}}=-4$ $\bullet^2$ ✓̸

Midpoint of AB (4, 1) $\bullet^3$ ✓

$y-1=-4(x-1)$ $\bullet^4$ ✓̸

$y+4x-5$

Part (b)

$y+4x-5=0$
 $y+2x+6=0$ $\bullet^5$ ✗ $\Rightarrow$ $y+2x=-6$
 $y+4x=-5$ $\bullet^6$ ✗

$\Rightarrow 2x=1,\ \ x=\frac{1}{2},\ y=-7$

$\bullet^5$ is a strategy mark. The correct strategy is to solve the **given equation** with the equation from part (a) simultaneously. $\bullet^5$ is not awarded as the given equation has not been used.

The equation obtained at stage $\bullet^4$, has been rearranged incorrectly in part (b). The next pd mark, $\bullet^6$, is therefore not awarded.

Question	Generic Scheme	Illustrative Scheme	Max Mark
2			
$\bullet^1$ ss	know to and differentiate	$\bullet^1$ $4x^3-6x^2$	
$\bullet^2$ ic	find gradient	$\bullet^2$ 8	
$\bullet^3$ pd	find y-coordinate	$\bullet^3$ 5	
$\bullet^4$ ic	state equation of tangent	$\bullet^4$ $y-5=8(x-2)$	**4**

Notes:

1. $\bullet^4$ is only available if an attempt has been made to find the gradient from differentiation **and** calculating the *y*-coordinate by substitution into the original equation.

Commonly Observed Responses:

Candidate A

$\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

using $y=mx+c$

$x=2,\ y=5,\ m=8$

$\Rightarrow 5=8\times 2+c$

$\Rightarrow c=-11$ $\bullet^4$ ✓

$y=8x-11$

Question		Generic Scheme	Illustrative Scheme	Max Mark
3	**a**			
		$\bullet^1$ ic interpret notation $\bullet^2$ pd a correct expression	$\bullet^1$ $f\left(x+3\right)$ stated or implied by $\bullet^2$ $\bullet^2$ $=\left(x+3\right)\left(x+2\right)+q$ **OR** $=\left(x+3\right)^2-\left(x+3\right)+q$ or equivalent	**2**

Notes:

1. Special Case: $\bullet^1$ is for substituting $(x+3)$ for x thus, treat $x+3(x+3-1)+q$ as bad form.

Commonly Observed Responses:

Candidate A	**Candidate B**
$f(g(x))=x+3(x+3-1)+q$ $\bullet^1$ ✓ $=x^2+5x+6+q$ $\bullet^2$ ✓ $\bullet^3$ ✓	$f(g(x))=x+3(x+3-1)+q$ $\bullet^1$ ✓ $=4x+6+q$ $\bullet^2$ X
Candidate C	**Candidate D**
$f(g(x))=x+3(x+3-1)+q$ $\bullet^1$ ✓ $=(x+3)^2-x+3+q$ $x^2+5x+6+q=0$ $\bullet^2$ ✓ $\bullet^3$ ✓	$f(g(x))=(x+3)(x+3-1)+q$ $\bullet^1$ ✓ $\bullet^2$ ✓ $=(x+3)^2-x+3+q$ $x^2+5x+12+q=0$ $\bullet^3$ X

Candidate E: using $g(f(x))$

part (a)

$g(f(x))=g(x(x-1)+q)$ $\bullet^1$ X

$=x(x-1)+q+3$ $\bullet^2$ ✓

Leading to …..

part (b)

$x^2-x+q+3=0$ $\bullet^3$ ✓ (eased)

$b^2-4ac=\left(-1\right)^2-4\times1\times(q+3)$ $\bullet^4$ ✓

$1-4q-12=0$ $\bullet^5$ ✓

$q=-\frac{11}{4}$ $\bullet^6$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
3 b			
	Method 1	**Method 1**	4
	$\bullet^3$ pd write in standard quadratic form	$\bullet^3$ $x^2+5x+6+q=0$	
	$\bullet^4$ ic use discriminant	$\bullet^4$ $b^2-4ac=5^2-4\times1\times(6+q)$	
	$\bullet^5$ pd simplify and equate to zero	$\bullet^5$ $\Rightarrow 25-24-4q=0$	
	$\bullet^6$ pd find value of *q*	$\bullet^6$ $q=\frac{1}{4}$	
	Method 2	**Method 2**	
	$\bullet^3$ pd write in standard quadratic form	$\bullet^3$ $x^2+5x+6+q=0$	
	$\bullet^4$ ic complete the square	$\bullet^4$ $\left(x+\frac{5}{2}\right)^2-\frac{25}{4}+6+q=0$	
	$\bullet^5$ pd equate to zero	$\bullet^5$ $-\frac{25}{4}+6+q=0$	
	$\bullet^6$ pd find value of *q*	$\bullet^6$ $q=\frac{1}{4}$	
	Method 3	**Method 3**	
	$\bullet^3$ pd write in standard quadratic form	$\bullet^3$ $f(g(x))=x^2+5x+6+q=0$	
	$\bullet^4$ ic geometric interpretation	$\bullet^4$ equal roots so touches *x*-axis at SP	
	$\bullet^5$ pd differentiates to obtain *x*	$\bullet^5$ $\Rightarrow \frac{dy}{dx}=2x+5=0$ $x=-\frac{5}{2}$	
	$\bullet^6$ pd find value of *q*	$\bullet^6$ $\frac{25}{4}-\frac{25}{2}+6+q=0$ $q=\frac{1}{4}$	

Notes:

2. Do not penalise the omission of ‘ $=0$ ’ at $\bullet^3$.
3. In Method 1 $a=1,\ b=5,\ c=6+q$ is sufficient for $\bullet^3$.
4. Candidates who assume ‘ $=0$ ’ and follow through to a correct value of *q*, $\bullet^6$ is still available. In Methods 1 and 2 ‘ $=0$ ’ must appear at $\bullet^4$ or $\bullet^5$ for $\bullet^5$ to be awarded.
5. If the expression obtained at $\bullet^3$ is not a quadratic then $\bullet^3$, $\bullet^4$, $\bullet^5$ and $\bullet^6$ are not available.

<table>
<tr><th>Question</th><th colspan="2">Generic Scheme</th><th colspan="2">Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td colspan="6">Throughout this question treat coordinates written as components, and vice versa, as bad form.</td></tr>
<tr><td>4 a</td><td colspan="4"></td><td></td></tr>
<tr><td>$\bullet^1$</td><td>pd</td><td>states coordinates of C</td><td>$\bullet^1$</td><td>C(11,12,6)</td><td></td></tr>
<tr><td>$\bullet^2$</td><td>pd</td><td>states coordinates of D</td><td>$\bullet^2$</td><td>$D(8,8,4)$</td><td>2</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. Accept $x = 11,\ y = 12$ and $z = 6$ for $\bullet^1$ and $x = 8,\ y = 8$ and $z = 4$ for $\bullet^2$.
2. For candidates who write the coordinates as Cartesian triples and omit brackets in both cases, $\bullet^2$ is not available.</td></tr>
<tr><td>4 b</td><td colspan="4"></td><td></td></tr>
<tr><td>$\bullet^3$</td><td>pd</td><td>finds $\overrightarrow{CB}$</td><td>$\bullet^3$</td><td>$\begin{pmatrix}0\\-8\\-4\end{pmatrix}$</td><td></td></tr>
<tr><td>$\bullet^4$</td><td>pd</td><td>finds $\overrightarrow{CD}$</td><td>$\bullet^4$</td><td>$\begin{pmatrix}-3\\-4\\-2\end{pmatrix}$</td><td>2</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">3. For candidates who find both $\overrightarrow{BC}$ and $\overrightarrow{DC}$, only $\bullet^4$ is available (repeated error).</td></tr>
<tr><td>4 c</td><td colspan="4">.</td><td></td></tr>
<tr><td>$\bullet^5$</td><td>ss</td><td>know to use scalar product applied to the correct angle</td><td>$\bullet^5$</td><td>$\cos B\hat{C}D = \frac{\overrightarrow{CB}.\overrightarrow{CD}}{|\overrightarrow{CB}||\overrightarrow{CD}|}$
stated or implied by $\bullet^9$</td><td></td></tr>
<tr><td>$\bullet^6$</td><td>pd</td><td>find scalar product</td><td>$\bullet^6$</td><td>40</td><td></td></tr>
<tr><td>$\bullet^7$</td><td>pd</td><td>find $|\overrightarrow{CB}|$</td><td>$\bullet^7$</td><td>$\sqrt{80}$</td><td></td></tr>
<tr><td>$\bullet^8$</td><td>pd</td><td>find $|\overrightarrow{CD}|$</td><td>$\bullet^8$</td><td>$\sqrt{29}$</td><td></td></tr>
<tr><td>$\bullet^9$</td><td>pd</td><td>find angle</td><td>$\bullet^9$</td><td>33·9°</td><td>5</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">4. $\bullet^5$ is not available for candidates who choose to evaluate an incorrect angle.
5. $\bullet^9$ accept 33·8 to 34 degrees or 0·59 to 0·6 radians.
6. If candidates do not attempt $\bullet^9$, then $\bullet^5$ is only available if the formula quoted relates to the labelling in the question.
7. $\bullet^9$ is only available as a result of using a valid strategy.
8. $\bullet^5$ is not available for candidates who write $\cos\theta = \frac{40}{\sqrt{80}\times\sqrt{29}}$. Some reference to the labelling of the diagram must be made within their solution to part (c), to indicate they are attempting to find the correct angle.</td></tr>
</table>

Commonly Observed Responses:

Candidate A: Cosine Rule $\cos B\hat{C}D = \dfrac{CB^2 + CD^2 - BD^2}{2 \times CB \times CD}$ $\bullet^5$ ✓ $CB = \sqrt{80},\ CD = \sqrt{29},\ BD = \sqrt{29}$ $\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✓ $33{\cdot}9^{\circ}$ or $0{\cdot}59$ radians $\bullet^9$ ✓	**Candidate B** $\cos B\hat{C}D = \dfrac{\overrightarrow{BC}.\overrightarrow{CD}}{\lvert\overrightarrow{BC}\rvert \times \lvert\overrightarrow{CD}\rvert}$ $\bullet^5$ ✗ $\overrightarrow{BC}.\overrightarrow{CD} = -40$ $\bullet^6$ ✓ $\lvert\overrightarrow{BC}\rvert = \sqrt{80}\ ,\ \lvert\overrightarrow{CD}\rvert = \sqrt{29}$ $\bullet^7$ ✓ $\bullet^8$ ✓ $146{\cdot}1^{\circ}$ or $2{\cdot}55$ radians $\bullet^9$ ✓
Candidate C $\cos B\hat{O}D = \dfrac{\overrightarrow{OB}.\overrightarrow{OD}}{\lvert\overrightarrow{OB}\rvert \times \lvert\overrightarrow{OD}\rvert}$ $\bullet^5$ ✗ $\overrightarrow{OB}.\overrightarrow{OD} = 128$ $\bullet^6$ ✓ $\lvert\overrightarrow{OB}\rvert = \sqrt{141}\ ,\ \lvert\overrightarrow{OD}\rvert = 12$ $\bullet^7$ ✓ $\bullet^8$ ✓ $26{\cdot}1^{\circ}$ or $0{\cdot}46$ radians $\bullet^9$ ✓	**Candidate D** $\cos C\hat{B}D = \dfrac{\overrightarrow{BC}.\overrightarrow{BD}}{\lvert\overrightarrow{BC}\rvert \times \lvert\overrightarrow{BD}\rvert}$ $\bullet^5$ ✗ $\overrightarrow{BC}.\overrightarrow{BD} = 40$ $\bullet^6$ ✓ $\lvert\overrightarrow{BC}\rvert = \sqrt{80}\ ,\ \lvert\overrightarrow{BD}\rvert = \sqrt{29}$ $\bullet^7$ ✓ $\bullet^8$ ✓ $33{\cdot}9^{\circ}$ or $0{\cdot}59$ radians $\bullet^9$ ✓
Candidate E $\cos B\hat{O}C = \dfrac{\overrightarrow{OB}.\overrightarrow{OC}}{\lvert\overrightarrow{OB}\rvert \times \lvert\overrightarrow{OC}\rvert}$ $\bullet^5$ ✗ $\overrightarrow{OB}.\overrightarrow{OC} = 181$ $\bullet^6$ ✓ $\lvert\overrightarrow{OB}\rvert = \sqrt{141}\ ,\ \lvert\overrightarrow{OC}\rvert = \sqrt{301}$ $\bullet^7$ ✓ $\bullet^8$ ✓ $28{\cdot}5^{\circ}$ or $0{\cdot}50$ radians $\bullet^9$ ✓	**Candidate F** $\cos B\hat{C}D = \dfrac{\overrightarrow{BC}.\overrightarrow{DC}}{\lvert\overrightarrow{BC}\rvert \times \lvert\overrightarrow{DC}\rvert}$ $\bullet^5$ ✓ this is an acceptable form for the scalar product.

Question		Generic Scheme			Illustrative Scheme		Max Mark
5							
		$\bullet^1$	ss	start to integrate	$\bullet^1$	$\frac{1}{1/2}(......)^{1/2}$	
		$\bullet^2$	pd	complete integration	$\bullet^2$	$...\times\frac{1}{3}$	
		$\bullet^3$	pd	process limits	$\bullet^3$	$\frac{2}{3}(3t+4)^{\frac{1}{2}}-\frac{2}{3}(3(4)+4)^{\frac{1}{2}}$	
		$\bullet^4$	pd	start to solve equation	$\bullet^4$	$(3t+4)^{\frac{1}{2}}=7$	
		$\bullet^5$	pd	solve for t	$\bullet^5$	$t = 15$	**5**

Notes:

1. $\bullet^3$ is awarded for correct substitution leading to $F(t)-F(4)$ where $F(x)$ is the candidates attempt
2. to integrate $(3x+4)^{-\frac{1}{2}}$. For substituting into the original function $\bullet^3$ is unavailable.
3. $\bullet^5$ is only available as a consequence of squaring both sides of an equation.
4. The integral obtained must contain a non integer power for $\bullet^4$ and $\bullet^5$ to be available.
5. Do not penalise the inclusion of '$+c$'.
6. Incorrect expansion of $(...)^{-1/2}$ at stage $\bullet^1$, only $\bullet^3$ is available as follow through. Incorrect expansion of $(...)^{1/2}$ at stage $\bullet^4$, $\bullet^4$ and $\bullet^5$ are not available.

Commonly Observed Responses:

Candidate A: Forgetting the $\frac{1}{3}$

$\left[2(3x+4)^{\frac{1}{2}}\right]_4^t=2$ $\bullet^1$ ✓ $\bullet^2$ ✗

$\left(2(3t+4)^{\frac{1}{2}}\right)-\left(2(3(4)+4)^{\frac{1}{2}}\right)=2$ $\bullet^3$ ✓

$(3t+4)^{\frac{1}{2}}=5$ $\bullet^4$ ✓

$t=7$ $\bullet^5$ ✓

Candidate B

$\left[\frac{1}{6}(3x+4)^{\frac{1}{2}}\right]_4^t=2$ $\bullet^1$ ✗ $\bullet^2$ ✓

$\left(\frac{1}{6}(3t+4)^{\frac{1}{2}}\right)-\left(\frac{1}{6}(3(4)+4)^{\frac{1}{2}}\right)=2$ $\bullet^3$ ✓

$(3t+4)^{\frac{1}{2}}=16$ $\bullet^4$ ✓

$t=84$ $\bullet^5$ ✓

Candidate C

$\left[\frac{(3x+4)^{\frac{1}{2}}}{1/2}\times3\right]_4^t=2$ $\bullet^1$ ✓ $\bullet^2$ ✗

$\left[\frac{2}{3}(3x+4)^{\frac{1}{2}}\right]_4^t=2$

$\left[\frac{2}{3}(3t+4)^{\frac{1}{2}}\right]-\left[\frac{2}{3}(3(4)+4)^{\frac{1}{2}}\right]=2$ $\bullet^3$ ✗

$(3t+4)^{\frac{1}{2}}=7$ $\bullet^4$ ✓

$t=15$ $\bullet^5$ ✓

Candidate D

$\left[-\frac{3}{2}(3x+4)^{-3/2}\right]_4^t=2$ $\bullet^1$ ✗ $\bullet^2$ ✗

$-\frac{3}{2}(3t+4)^{-3/2}-\left(-\frac{3}{2}\times16^{-3/2}\right)=2$ $\bullet^3$ ✓

$(3t+4)^{3/2}=-\frac{192}{253}$ $\bullet^4$ ✓

decimal equivalent not accepted

$t=-1\cdot056$ $\bullet^5$ ✓

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Question	Generic Scheme	Illustrative Scheme	Max Mark
6			
	•1 ss use correct double angle formula	•1 $\sin x-2\left(1-2\sin^2 x\right)$ stated or implied by •2	
	•2 ss arrange in standard quadratic form	•2 $4\sin^2 x+\sin x-3=0$	
	•3 ss start to solve	•3 $\left(4\sin x-3\right)\left(\sin x+1\right)=0$	
		OR	
		$\dfrac{-1\pm\sqrt{(1)^2-4\times4\times(-3)}}{2\times4}$	
	•4 ic reduce to equations in $\sin x$ only	•4 $\sin x=\dfrac{3}{4}$ and $\sin x=-1$	
	•5 pd process to find solutions in given domain	•5 $0\cdot848$, $2\cdot29$ and $\dfrac{3\pi}{2}$	**5**
		OR	
		•4 $\sin x=\dfrac{3}{4}$ and $x=0\cdot848,\ 2\cdot29$	
		•5 $\sin x=-1$, and $x=\dfrac{3\pi}{2}$	

Notes:

1. •1 is not available for simply stating $\cos 2A=1-2\sin^2 A$ with no further working.
2. In the event of $\cos^2 x-\sin^2 x$ or $2\cos^2 x-1$ being substituted for $\cos 2x$, •1 cannot be awarded until the equation reduces to a quadratic in $\sin x$.
3. Substituting $1-2\sin^2 A$ or $1-2\sin^2\alpha$ for $\cos 2\alpha$ at •1 stage should be treated as bad form provided the equation is written in terms of x at stage •2. Otherwise, •1 is not available.
4. ‘$=0$’ must appear by •3 stage for •2 to be awarded. However , for candidates using the quadratic formula to solve the equation, ‘$=0$’ must appear at •2 stage for •2 to be awarded.
5. Candidates may express the equation obtained at •2 in the form $4s^2+s-3=0$ or $4x^2+x-3=0$. In these cases, award •3 for $(4s-3)(s+1)=0$ or $(4x-3)(x+1)=0$. However, •4 is only available if $\sin x$ appears explicitly at this stage.
6. •4 and •5 are only available as a consequence of solving a quadratic equation.
7. •3 , •4 and •5 are not available for any attempt to solve a quadratic written in the form $ax^2+bx=c$.
8. •5 is not available to candidates who work in degrees and do not convert their solutions into radian measure.
9. $\sin x+4\sin^2 x-3=0$ does not gain •2, unless •3 is awarded.

Commonly Observed Responses:

Candidate A

$\bullet^1$ ✓ $\bullet^2$ ✓

$(4s-3)(s+1)=0$ $\bullet^3$ ✓

$s=\frac{3}{4},\ s=-1$ $\bullet^4$ ✗

$x=0{\cdot}848,\ 2{\cdot}29$ and $\frac{3\pi}{2}$ $\bullet^5$ ✓

Candidate B

$\bullet^1$ ✓

$4\sin^2 x+\sin x-3=0$ $\bullet^2$ ✓

$5\sin x-3=0$ $\bullet^3$ ✗

$\sin x=\frac{3}{5}$ $\bullet^4$ ✗

$x=0{\cdot}644,\ 2{\cdot}50$ $\bullet^5$ $\cancel{\cancel{\checkmark}}$

Candidate C

$\bullet^1$ ✓

$\sin x-2(1-2\sin^2 x)=1$

$\sin x-2+4\sin^2 x=1$

$4\sin^2 x+\sin x=3$ $\bullet^2$ $\cancel{\cancel{\checkmark}}$

$\sin x(4\sin x+1)=3$ $\bullet^3$ $\cancel{\cancel{\checkmark}}$

$\sin x=3,\quad 4\sin x+1=3$ $\bullet^4$ ✗

no solution, $\sin x=\frac{1}{2}$

$x=\frac{\pi}{6},\ \frac{5\pi}{6}$ $\bullet^5$ $\cancel{\cancel{\checkmark}}$

Candidate D

$\bullet^1$ ✓

$\sin x-2(1-2\sin^2 x)=1$

$4\sin^2 x+\sin x-3=0$ $\bullet^2$ ✓

$4\sin^2 x+\sin x=3$

$\sin x(4\sin x+1)=3$ $\bullet^3$ $\cancel{\cancel{\checkmark}}$

$\sin x=3,\quad 4\sin x+1=3$ $\bullet^4$ ✗

no solution, $\sin x=\frac{1}{2}$

$x=\frac{\pi}{6},\ \frac{5\pi}{6}$ $\bullet^5$ $\cancel{\cancel{\checkmark}}$

Candidate E: Reading $\cos 2x$ as $\cos^2 x$

$\sin x-2\cos^2 x=1$ $\bullet^1$ ✗

$\sin x-2(1-\sin^2 x)=1$

$2\sin^2 x+\sin x-3=0$ $\bullet^2$ $\cancel{\checkmark}$

$(2\sin x+3)(\sin x-1)=0$ $\bullet^3$ $\cancel{\checkmark}$

$\sin x=-\frac{3}{2},\quad \sin x=1$ $\bullet^4$ $\cancel{\checkmark}$

no solution, $x=\frac{\pi}{2}$ $\bullet^5$ $\cancel{\cancel{\checkmark}}$

Question		Generic Scheme		Illustrative Scheme	Max Mark
7	**a**				
•1		ss	know to and find intersection of line and curve	•1 $2x = 6x - x^2 \Rightarrow x = 0, x = 4$	
•2		ic	use " upper – lower"	•2 $\int \left(\left(6x - x^2 \right) - 2x \right) dx$	
•3		pd	integrate	•3 $2x^2 - \frac{1}{3}x^3$	
•4		pd	substitute limits and evaluate	•4 $10\frac{2}{3}$	
•5		pd	evaluate area developed	•5 $10\frac{2}{3} \times 300 = 3200\,\text{m}^2$	**5**

Notes:

1. '0' appearing as the lower limit of the integral is sufficient evidence for $x = 0$ at •1 stage.
2. •5 is only available as a consequence of multiplying an **exact** answer at •4 stage.
3. The omission of dx at •2 should not be penalised.
4. Where a candidate differentiates one or both terms •3, •4 and •5 are unavailable.
5. Do not penalise the inclusion of '$+ c$'.
6. Accept $\int \left(4x - x^2 \right) dx$ for •2.

Commonly Observed Responses:

Candidate A

$\int_0^4 \left(2x-\left(6x-x^2\right)\right)dx$ $\bullet^2$ X

$=\frac{1}{3}x^3-2x^2$ $\bullet^3$ ✓

$=-10\frac{2}{3}$ cannot be negative so $=10\frac{2}{3}$ $\bullet^4$ X however .... $=-10\frac{2}{3}$ so Area $=10\frac{2}{3}$ $\bullet^4$ ✓

Area = 3200m^2 $\bullet^5$ ✓

Candidate B

$2x=6x-x^2 \Rightarrow x=0,\ 4$ $\bullet^1$ ✓

Shaded area

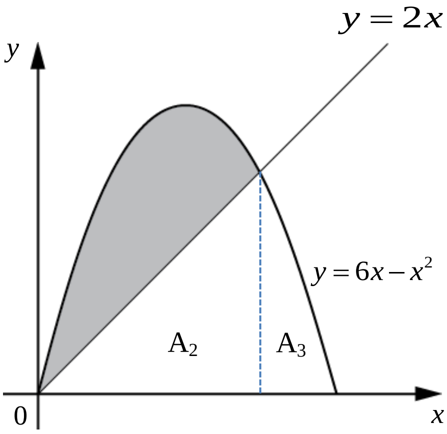
= area under parabola – (A_2 + A_3)

$=\int_0^6 \left(6x-x^2\right)dx-\left[A_2+\int_4^6 (6x-x^2)dx\right]$ $\bullet^2$ ✓

Stated or implied by $\bullet^4$

Area under parabola = 36, A_2 = 16 and A_3 = $\frac{28}{3}$ $\bullet^3$ ✓

Shaded area $=36-\left(16+\frac{28}{3}\right)=\frac{32}{3}$ $\bullet^4$ ✓



Candidate C

Part (a)

$x=0,\ x=6$ $\bullet^1$ X

$\int\left(\left(6x-x^2\right)-2x\right)dx$ $\bullet^2$ ✓

$\left[2x^2-\frac{1}{3}x^3\right]_0^6$ $\bullet^3$ ✓

$\left(2\times6^2-\frac{1}{3}\times6^3\right)-\left(0\right)=0$ $\bullet^4$ X

$\Rightarrow$ Area $=0\times300=0$ m^2 $\bullet^5$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 b			
	$\bullet^6$ ss set derivative to 2	$\bullet^6$ $6-2x=2$	
	$\bullet^7$ pd find point of contact	$\bullet^7$ $x = 2,\ y = 8$	
	$\bullet^8$ pd find equation of road	$\bullet^8$ $y = 2x + 4$	
	$\bullet^9$ ss find correct integral	$\bullet^9$ $\left[\left(x^2+4x\right)-\left(3x^2-\frac{1}{3}x^3\right)\right]_0^2$	
	$\bullet^{10}$ ic calculate area	$\bullet^{10}$ 800m^2	5

Notes:

6. For candidates who omit ‘ m^2 ’ at both $\bullet^5$ and $\bullet^{10}$ stages, $\bullet^{10}$ is not available.
7. Candidates who arrive at an incorrect equation at $\bullet^8$, or produce an equation ex nihilo, must use an equation of the form $y=2x+c$ with $c>0$, for $\bullet^9$ and $\bullet^{10}$ to be available.
8. $y=2x+4$ must appear explicitly or as part of the integrand for $\bullet^8$ to be awarded.
9. $\bullet^{10}$ is only available as a result of a valid strategy at the $\bullet^9$ stage, ie $\int(\text{line})-(\text{quadratic})$ **and** lower limit = 0 and upper limit < 3.

Commonly Observed Responses:

Candidate D: Alternative Method

Line has equation of the form $y=2x+c$, $y=2x+c$ and $y=6x-x^2$

intersect where $x^2-4x+c=0$ $\bullet^6$ ✓

tangency $\Rightarrow$ 1 point of intersection

$\Rightarrow b^2-4ac=0$ $\bullet^7$ ✓

$16-4c=0$ $\bullet^8$ ✓

$c=4$

Continue as above.

Question		Generic Scheme	Illustrative Scheme	Max Mark
8				
		$\bullet^1$ pd correct values	$\bullet^1$ $g=-p$, $f=-2p$, $c=3p+2$	
		$\bullet^2$ ss substitute and rearrange	$\bullet^2$ $5p^2-3p-2$	
		$\bullet^3$ ic knowing condition	$\bullet^3$ $g^2+f^2-c>0$	
		$\bullet^4$ pd factorise and solve	$\bullet^4$ $(5p+2)(p-1)=0 \Rightarrow p=-\frac{2}{5}, p=1$	
		$\bullet^5$ ic correct range	$\bullet^5$ $p<-\frac{2}{5},\ p>1$	**5**

Notes:

1. Candidates who state the coordinates of the centre, $(p,2p)$ and state the radius, $r=\sqrt{...-(3p+2)}$ gain $\bullet^1$.
2. Accept $(-p)^2+(-2p)^2-(3p+2)$ or $p^2+(2p)^2-(3p+2)$. If brackets are omitted $\bullet^1$ may only be awarded if subsequent working is correct.
3. Do not accept $(-p)^2+(2p)^2-(3p+2)$ or $(p)^2+(-2p)^2-(3p+2)$ for $\bullet^1$.
4. Do not accept $g^2+f^2-c\geq 0$ for $\bullet^3$.
5. For a candidate who uses $c=2$ and follows through to get $p<-\sqrt{\frac{2}{5}},\ p>\sqrt{\frac{2}{5}}$, award $\bullet^2$, $\bullet^3$ and $\bullet^5$.
6. Evidence for $\bullet^3$ may appear at $\bullet^5$ stage.
7. $\bullet^4$ and $\bullet^5$ can only be awarded for solving a quadratic inequation.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$g=-2p,\ f=-4p,\ c=3p+2$	$\bullet^1$ ✗	$(x-p)^2-p^2+(y-2p)^2-4p^2+3p+2=0$	
$20p^2-3p-2$	$\bullet^2$ ✓	$(x-p)^2+(y-2p)^2$	$\bullet^1$ ✓
$g^2+f^2-c>0$	$\bullet^3$ ✓	$=5p^2-3p-2$	$\bullet^2$ ✓
$(4p+1)(5p-2)=0 \Rightarrow p=-\frac{1}{4}, p=\frac{2}{5}$	$\bullet^4$ ✓	$5p^2-3p-2>0$	$\bullet^3$ ✓
$p<-\frac{1}{4},\ p>\frac{2}{5}$	$\bullet^5$ ✓	$(5p+2)(p-1)>0$	$\bullet^4$ ✓
		$p<-\frac{2}{5},\ p>1$	$\bullet^5$ ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
9	a			
		•1 ss know to differentiate	•1 $a=v'(t)$	
		•2 pd differentiates trig. function	•2 $-8\sin\left(2t-\frac{\pi}{2}\right)\ldots\ldots$	
		•3 pd applies chain rule	•3 $\ldots\ldots\times 2$ and complete $a(t)=-16\sin\left(2t-\frac{\pi}{2}\right)$	3

Commonly Observed Responses:

Candidate A: Alternative Method

Part (a)

$v(t)=8\cos\left(2t-\frac{\pi}{2}\right)=8\sin 2t$

$v'(t)=\ldots.$ •1 ✓

$=8\cos 2t\ldots$

•2 ✓

$=\ldots\ldots\times 2$ •3 ✓

Part (b)

$v'(10)=16\cos 20=6\cdot 53$ •4 ✓

$>0,\ \Rightarrow$ velocity is increasing •5 ✓

Part (c)

$s(t)=\int v(t)dt$ •6 ✓

$s(t)=-4\cos 2t+c$ •7 ✓

$4=-4+c\Rightarrow c=8$

$\Rightarrow s(t)=-4\cos 2t+8$ •8 ✓

or $\Rightarrow s(t)=8-4\cos 2t$

Candidate B: Candidates who misinterpret the process for rate of change.

Part (a)

$a(t)=\int 8\cos\left(2t-\frac{\pi}{2}\right)dt$

$=4\sin\left(2t-\frac{\pi}{2}\right)+c$

Wrong process award $0/3$

Part (b)

If $t=10$, $a=4\sin\left(20-\frac{\pi}{2}\right)+c$

$=-1\cdot 63+c$

Cannot evaluate award $0/2$

Part (c)

$s=v'(t)$

$s(t)=-16\sin\left(2t-\frac{\pi}{2}\right)$

Award $2/3$

Candidate C

Part (a)

$a=v'(t)$ or equivalent •1

$a=4\sin\left(2t-\frac{\pi}{2}\right)$ •2 ✗ •3 ✗

Part (b)

$a(10)=4\sin\left(20-\frac{\pi}{2}\right)=-1\cdot 63$ •4

<0, So decreasing •5 ⍻

Only as a consequence of •1 in part (a)

Question				Generic Scheme	Illustrative Scheme	Max Mark
9	b					
		$\bullet^4$	ss	know to and evaluate $a(10)$	$\bullet^4$ $a(10)=6\cdot53$	2
		$\bullet^5$	ic	interpret result	$\bullet^5$ $a(10)>0$ therefore increasing	

Notes:

1. $\bullet^5$ is available only as a consequence of substituting into a derivative.
2. $\bullet^4$ and $\bullet^5$ are not available to candidates who work in degrees.
3. $\bullet^2$ and $\bullet^3$ may be awarded if they appear in the working for 9(b). However, $\bullet^1$ requires a clear link between acceleration and $v'(t)$.

Question				Generic Scheme	Illustrative Scheme	Max Mark
9	c					
		$\bullet^6$	ic	know to integrate	$\bullet^6$ $s(t)=\int v(t)\,dt$	3
		$\bullet^7$	pd	integrate correctly	$\bullet^7$ $s(t)=4\sin\left(2t-\frac{\pi}{2}\right)+c$	
		$\bullet^8$	ic	determine constant and complete	$\bullet^8$ $c=8 \text{so} s(t)=4\sin\left(2t-\frac{\pi}{2}\right)+8$	

Notes:

4. $\bullet^7$ and $\bullet^8$ are not available to candidates who work in degrees. However, accept $\int 8\cos(2t-90)\,dt$ for $\bullet^6$.

[END OF MARKING INSTRUCTIONS]