H

National
Qualifications
2018

2018 Mathematics

Higher - Paper 1

Finalised Marking Instructions

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$ Vertical: •5 $x = 2$ and $y = 5$

•6 $y = 5$ and $y = -7$ •6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	$\bullet^1$ find mid-point of PQ	$\bullet^1$ $(1,2)$	**3**
	$\bullet^2$ find gradient of median	$\bullet^2$ 2	
	$\bullet^3$ determine equation of median	$\bullet^3$ $y=2x$	

Notes:

1. $\bullet^2$ is only available to candidates who use a midpoint to find a gradient.
2. $\bullet^3$ is only available as a consequence of using the mid-point and the point R, or any other point which lies on the median, eg $(2,4)$.
3. At $\bullet^3$ accept any arrangement of a candidate's equation where constant terms have been simplified.
4. $\bullet^3$ is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - Perpendicular Bisector of PQ

$M_{PQ}(1,2)$	$\bullet^1$ ✓
$m_{PQ}=-\frac{2}{3} \Rightarrow m_{\perp}=\frac{3}{2}$	$\bullet^2$ ✗
$2y=3x+1$	$\bullet^3$ $\boxed{\checkmark\ 2}$

For other perpendicular bisectors award 0/3

Candidate B - Altitude through R

$m_{PQ}=-\frac{2}{3}$	$\bullet^1$ ^
$m_{\perp}=\frac{3}{2}$	$\bullet^2$ ✗
$2y=3x+3$	$\bullet^3$ $\boxed{\checkmark\ 2}$

Candidate C - Median through P

$M_{QR}(3\cdot5,3)$	$\bullet^1$ ✗
$m_{PM}=-\frac{2}{11}$	$\bullet^2$ $\boxed{\checkmark\ 1}$
$11y+2x=40$	$\bullet^3$ $\boxed{\checkmark\ 2}$

Candidate D - Median through Q

$M_{PR}(0\cdot5,5)$	$\bullet^1$ ✗
$m_{QM}=-\frac{10}{7}$	$\bullet^2$ $\boxed{\checkmark\ 1}$
$7y+10x=40$	$\bullet^3$ $\boxed{\checkmark\ 2}$

Question	Generic scheme	Illustrative scheme	Max mark
2.	**Method 1**	**Method 1**	3
	•¹ equate composite function to x	•¹ $g\left(g^{-1}(x)\right)=x$	
	•² write $g(g^{-1}(x))$ in terms of $g^{-1}(x)$	•² $\frac{1}{5}g^{-1}(x)-4=x$	
	•³ state inverse function	•³ $g^{-1}(x)=5(x+4)$	
	Method 2	**Method 2**	
	•¹ write as $y=\frac{1}{5}x-4$ and start to rearrange	•¹ $y+4=\frac{1}{5}x$	
	•² express x in terms of y	•² eg $x=5(y+4)$ or $x=\frac{(y+4)}{\frac{1}{5}}$	
	•³ state inverse function	•³ $g^{-1}(x)=5(x+4)$	
	Method 3	**Method 3**	
	•¹ interchange variables	•¹ $x=\frac{1}{5}y-4$	
	•² express y in terms of x	•² eg $y=5(x+4)$ or $y=\frac{(x+4)}{\frac{1}{5}}$	
	•³ state inverse function	•³ $g^{-1}(x)=5(x+4)$	

Notes:

1. $y=5(x+4)$ does not gain •³.
2. At •³ stage, accept g^{-1} written in terms of any dummy variable eg $g^{-1}(y)=5(y+4)$.
3. $g^{-1}(x)=5(x+4)$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$x \rightarrow \frac{1}{5}x \rightarrow \frac{1}{5}x-4=g(x)$

$\div 5 \rightarrow -4$

$\therefore\ +4 \rightarrow \times 5$ •¹ ✓ awarded for knowing to perform inverse operations in reverse order

$5(x+4)$ •² ✓

$g^{-1}(x)=5(x+4)$ •³ ✓

Candidate B - BEWARE	**Candidate C**
$g'(x)=\ldots$ •³ ✗	$g^{-1}(x)=5x+4$ **with no working** **Award 0/3**

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ start to differentiate $\bullet^2$ complete differentiation $\bullet^3$ evaluate derivative	$\bullet^1$ $-3\sin 2x\ldots$ stated or implied by $\bullet^2$ $\bullet^2$ $\ldots\times 2$ $\bullet^3$ $-3\sqrt{3}$	**3**

Notes:

1. Ignore the appearance of $+c$ at any stage.
2. $\bullet^3$ is available for evaluating an attempt at finding the derivative at $\frac{\pi}{6}$.
3. For $h'\left(\frac{\pi}{6}\right) = 3\cos\left(2\times\frac{\pi}{6}\right) = \frac{3}{2}$ award 0/3.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$-3\sin 2x\ldots$	$\bullet^1$ ✓	$3\sin 2x\ldots$	$\bullet^1$ ✗	$3\sin 2x\ldots$	$\bullet^1$ ✗
$\ldots\times\frac{1}{2}$	$\bullet^2$ ✗	$\ldots\times 2$	$\bullet^2$ ✓	$\ldots\times\frac{1}{2}$	$\bullet^2$ ✗
$-\frac{3\sqrt{3}}{4}$	$\bullet^3$ ✓ 1	$3\sqrt{3}$	$\bullet^3$ ✓ 1	$\frac{3\sqrt{3}}{4}$	$\bullet^3$ ✓ 1

Candidate D		Candidate E		Candidate F	
$\pm 6\cos 2x$	$\bullet^1$ ✗	$\pm 3\cos 2x\ldots$	$\bullet^1$ ✗	$6\sin 2x$	$\bullet^1$ ✗
	$\bullet^2$ ✗	$\ldots\times 2$	$\bullet^2$ ✓ 1		$\bullet^2$ ✓
± 3	$\bullet^3$ ✓ 1	± 3	$\bullet^3$ ✓ 1	$3\sqrt{3}$	$\bullet^3$ ✓ 1

Question	Generic scheme	Illustrative scheme	Max mark
4.	$\bullet^1$ state centre of circle $\bullet^2$ find gradient of radius $\bullet^3$ state gradient of tangent $\bullet^4$ state equation of tangent	$\bullet^1$ $(6,3)$ $\bullet^2$ -4 $\bullet^3$ $\frac{1}{4}$ $\bullet^4$ $y=\frac{1}{4}x-7$	4

Notes:

1. Accept $-\frac{8}{2}$ for $\bullet^2$.
2. The perpendicular gradient must be simplified at the $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be available.
3. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient.
4. At $\bullet^4$ accept $y-\frac{1}{4}x+7=0$, $4y=x-28$, $x-4y-28=0$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
5.	(a)	$\bullet^1$ state ratio explicitly	$\bullet^1$ $4:1$	**1**

Notes:

1. The only acceptable variations for $\bullet^1$ must be related explicitly to AB and BC.
   For $\frac{\text{BC}}{\text{AB}}=\frac{1}{4}$, $\frac{\text{AB}}{\text{BC}}=\frac{4}{1}$ or $\text{BC}:\text{AB}=1:4$ award 1/1.
2. For $\text{BC}=\frac{1}{4}\text{AB}$ award 0/1.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	$\bullet^2$ state value of t	$\bullet^2$ 8	**1**

Notes:

3. The answer to part (b) **must** be consistent with a ratio stated in part (a) unless a valid strategy which does not require the use of their ratio from part (a) is used.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$1:4$	$\bullet^1$ ✘	$1:4$	$\bullet^1$ ✘
$t=8$	$\bullet^2$ ✘	$t=5$	$\bullet^2$ $\boxed{✓ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
6.	•1 apply $m\log_5 x = \log_5 x^m$	•1 $\log_5 8^{\frac{1}{3}}$	3
	•2 apply $\log_5 x - \log_5 y = \log_5 \frac{x}{y}$	•2 $\log_5 \left(\frac{250}{8^{\frac{1}{3}}} \right)$	
	•3 evaluate log	•3 3	

Notes:

1. Each line of working must be equivalent to the line above within a valid strategy, however see Candidate B.
2. Do not penalise the omission of the base of the logarithm at •1 or •2.
3. For '3' with no working award 0/3.

Commonly Observed Responses:

Candidate A		Candidate B
$\log_5 250 - \log_5 \frac{8}{3}$	•1 ✗	$\frac{1}{3}\log_5 (250 \div 8)$
$\log_5 \frac{250}{\frac{8}{3}}$	•2 $\boxed{✓ 1}$	$\frac{1}{3}\log_5 \frac{125}{4}$
$\log_5 \frac{375}{4}$	•3 $\boxed{✓ 2}$	$\log_5 \left(\frac{125}{4} \right)^{\frac{1}{3}}$ Award 1/3 $\boxed{✓ 1}$ ✗ ^
		•1 is awarded for the final two lines of working

Question	Generic scheme	Illustrative scheme	Max mark
7. (a)	$\bullet^1$ state coordinates of P	$\bullet^1$ $(0,5)$	**1**
Notes:			
1. Accept '$x=0,\ y=5$'. 2. '$y=5$' alone or '5' does not gain $\bullet^1$.			
Commonly Observed Responses:			
(b)	$\bullet^2$ differentiate $\bullet^3$ calculate gradient $\bullet^4$ state equation of tangent	$\bullet^2$ $3x^2-6x+2$ $\bullet^3$ 2 $\bullet^4$ $y=2x+5$	**3**
Notes:			
3. At $\bullet^4$ accept $y-2x=5$, $2x-y+5=0$, $y-5=2x$ or any other rearrangement of the equation where the constant terms have been simplified. 4. $\bullet^4$ is only available if an attempt has been made to find the gradient from differentiation.			
Commonly Observed Responses:			

Question		Generic scheme	Illustrative scheme	Max mark
7.	(c)	•5 set $y_{\text{line}} = y_{\text{curve}}$ and arrange in standard form	•5 $x^3 - 3x^2 = 0$	4
		•6 factorise	•6 $x^2(x-3)$	
		•7 state x-coordinate of Q	•7 3	
		•8 calculate y-coordinate of Q	•8 11	

Notes:

5. •5 is only available if '$=0$' appears at either •5 or •6 stage.
6. •7 and •8 are only available as a consequence of solving a cubic equation and a linear equation simultaneously.
7. For an answer of $(3,11)$ with no working award 0/4.
8. For an answer of $(3,11)$ verified in both equations award 3/4.
9. For an answer of $(3,11)$ verified in both equations along with a statement such as 'same point on both line and curve so Q is $(3,11)$' award 4/4.
10. For candidates who work with a derivative, no further marks are available.
11. $x=3$ must be supported by valid working for •7 and •8 to be awarded.

Commonly Observed Responses:

Candidate A

$x^3 - 3x^2 = 0$	•5 ✓	
$x-3=0$	•6 ✓	
$x=3$	•7 ✓	
$y=11$	•8 ✓	

Dividing by x^2 is valid since $x \neq 0$ at •6

Question	Generic scheme	Illustrative scheme	Max mark
8.	•¹ determine the gradient of the line •² determine the angle	•¹ $m=\sqrt{3}$ or $\tan\theta=\sqrt{3}$ •² 60° or $\frac{\pi}{3}$	**2**

Notes:

1. Do not penalise the omission of units at •².
2. For 60° or $\frac{\pi}{3}$ without working award 2/2.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$y=\sqrt{3}x+5$	Ignore incorrect processing of the constant term	$m=\sqrt{3}$	•¹ ✓
		$\theta=\tan\sqrt{3}$	•² ✗
$m=\sqrt{3}$	•¹ ✓	$\theta=60°$	
60°	•² ✓	Stating $\tan$ rather than $\tan^{-1}$ See general marking principle (g)	

Question		Generic scheme	Illustrative scheme	Max mark
9.	(a)	•[1] identify pathway	•[1] $-\mathbf{t}+\mathbf{u}$	1

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•[2] state an appropriate pathway	•[2] eg $\frac{1}{2}\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{AD}$ stated or implied by •[3]	2
		•[3] express pathway in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$	•[3] $-\frac{1}{2}\mathbf{t}-\frac{1}{2}\mathbf{u}+\mathbf{v}$	

Notes:

1. There is no need to simplify the expression at •[3]. Eg $\frac{1}{2}(-\mathbf{t}+\mathbf{u})-\mathbf{u}+\mathbf{v}$.
2. •[3] is only available for using a valid pathway.
3. The expression at •[3] must be consistent with the candidate's expression at •[1].
4. If the pathway in •[1] is given in terms of a single vector $\mathbf{t}$, $\mathbf{u}$ or $\mathbf{v}$, then •[3] is not available.

Commonly Observed Responses:

Candidate A

$\overrightarrow{MD}=-\frac{1}{2}\mathbf{t}+\mathbf{v}-\mathbf{u}$ •[2] ^ •[3] ✗

Question	Generic scheme	Illustrative scheme	Max mark
10.	•¹ know to and integrate one term •² complete integration •³ substitute for x and y •⁴ state equation	•¹ eg $2x^3 \ldots$ •² eg $\ldots -\frac{3}{2}x^2+4x+c$ •³ $14=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$ •⁴ $y=2x^3-\frac{3}{2}x^2+4x-4$ stated explicitly	4

Notes:

1. For candidates who make no attempt to integrate to find y in terms of x award 0/4.
2. For candidates who omit $+c$, only •¹ is available.
3. Candidates must attempt to integrate both terms containing x for •³ and •⁴ to be available. See Candidate B.
4. For candidates who differentiate any term, •² •³ and •⁴ are not available.
5. •⁴ is not available for ‘ $f(x)=\ldots$ ’.
6. Candidates must simplify coefficients in **their** final line of working for the last mark available in that line of working to be awarded.

Commonly Observed Responses:

Candidate A		**Candidate B - partial integration**	
$y=2x^3-\frac{3}{2}x^2+4x+c$	•¹ ✓ •² ✓	$y=2x^3-\frac{3}{2}x^2+4+c$	•¹ ✓ •² ✗
$y=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$		$14=2(2)^3-\frac{3}{2}(2)^2+4+c$	•³ $\boxed{✓ 1}$
$c=-4$	•³ ✓ substitution for y implied by $c=-4$ •⁴ ^	$c=0$	
		$y=2x^3-\frac{3}{2}x^2+4$	•⁴ $\boxed{✓ 1}$

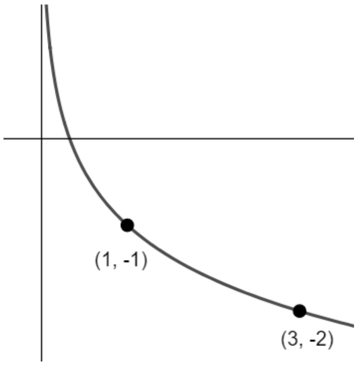
Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•[1] curve reflected in x-axis and translated 1 unit vertically •[2] accurate sketch	•[1] a generally decreasing curve above the x-axis for $1<x<3$ •[2] asymptote at $x=0$ **and** passing through $(3,0)$ **and** continuing to decrease for $x\geq 3$	**2**

Notes:

1. For any attempt which involves a horizontal translation or reflection in the y-axis award 0/2.
2. For a single transformation award 0/2.
3. For any attempt involving a reflection in the line $y=x$ award 0/2

Commonly Observed Responses:

Candidate A



Award 1/2

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•[3] set ‘ $y=y$ ’ •[4] start to solve •[5] state x coordinate	•[3] $\log_3 x = 1-\log_3 x$ •[4] $\log_3 x = \frac{1}{2}$ or $\log_3 x^2 = 1$ •[5] $\sqrt{3}$ or $3^{\frac{1}{2}}$	**3**

Notes:

4. •[3] may be implied by $\log_3 x = \frac{1}{2}$ from symmetry of the curves.
5. Do not penalise the omission of the base of the logarithm at •[3] or •[4].
6. For a solution which equates a to $\log_3 a$, the final mark is not available.
7. If a candidate considers and then does not discard $-\sqrt{3}$ in their final answer, •[5] is not available.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	$\bullet^1$ find components	$\bullet^1$ $\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	1

Notes:

1. Accept $6\mathbf{i}-3\mathbf{j}+(4+p)\mathbf{k}$ for $\bullet^1$.
2. Do not accept $\begin{pmatrix} 6\mathbf{i} \\ -3\mathbf{j} \\ (4+p)\mathbf{k} \end{pmatrix}$ or $6\mathbf{i}-3\mathbf{j}+4\mathbf{k}+p\mathbf{k}$ for $\bullet^1$. However $\bullet^2$, $\bullet^3$ and $\bullet^4$ are still available.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	$\bullet^2$ find an expression for magnitude	$\bullet^2$ $\sqrt{6^2+(-3)^2+(4+p)^2}$	3
	$\bullet^3$ start to solve	$\bullet^3$ $45+(4+p)^2=49 \Rightarrow (4+p)^2=4$ or $p^2+8p+12=0$	
	$\bullet^4$ find values of p	$\bullet^4$ $p=-2,\ p=-6$	

Notes:

3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{6^2+-3^2+(4+p)^2}$ or $\sqrt{6^2-3^2+(4+p)^2}$ leading to $\sqrt{45+(4+p)^2}$, $\bullet^2$ is awarded.
4. $\bullet^4$ is only available for two distinct values of p.

Commonly Observed Responses:

Candidate A		Candidate B	
$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓	$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓
$\sqrt{6^2-3^2+(4+p)^2}$	$\bullet^2$ ✗	$\sqrt{6^2+(-3)^2+p^2}$	$\bullet^2$ ✗
$27+(4+p)^2=49$		$45+p^2=49$	$\bullet^3$ $\boxed{\checkmark\ 2}$
$(4+p)^2=22$	$\bullet^3$ $\boxed{\checkmark\ 1}$	$p=\pm 2$	$\bullet^4$ $\boxed{\checkmark\ 1}$
$p=-4\pm\sqrt{22}$	$\bullet^4$ $\boxed{\checkmark\ 1}$		

Question	Generic scheme	Illustrative scheme	Max mark
13. (a) (i)	•¹ find the value of $\cos x$ •² substitute into the formula for $\sin 2x$ •³ simplify	•¹ $\frac{\sqrt{7}}{\sqrt{11}}$ stated or implied by •² •² $2\times\frac{2}{\sqrt{11}}\times\frac{\sqrt{7}}{\sqrt{11}}$ •³ $\frac{4\sqrt{7}}{11}$	3
(ii)	•⁴ evaluate $\cos 2x$	•⁴ $\frac{3}{11}$	1

Notes:

1. Where a candidate substitutes an incorrect value for $\cos x$ at •², •² may be awarded if the candidate has previously stated this incorrect value or it can be implied by a diagram.
2. •³ is only available as a consequence of substituting into a valid formula at •².
3. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•⁵ expand using the addition formula •⁶ substitute in values •⁷ simplify	•⁵ $\sin 2x\cos x+\cos 2x\sin x$ stated or implied by •⁶ •⁶ $\frac{4\sqrt{7}}{11}\times\frac{\sqrt{7}}{\sqrt{11}}+\frac{3}{11}\times\frac{2}{\sqrt{11}}$ •⁷ $\frac{34}{11\sqrt{11}}$	3

Notes:

4. For any attempt to use $\sin(2x+x)=\sin 2x+\sin x$, •⁵ •⁶ and •⁷ are not available

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
14.	•[1] write in integrable form	•[1] $(2x+9)^{-\frac{2}{3}}$	5
	•[2] start to integrate	•[2] $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\ldots$	
	•[3] complete integration	•[3] $\ldots\times\dfrac{1}{2}$	
	•[4] process limits	•[4] $\dfrac{3}{2}(2(9)+9)^{\frac{1}{3}}-\dfrac{3}{2}(2(-4)+9)^{\frac{1}{3}}$	
	•[5] evaluate integral	•[5] 3	

Notes:

1. For candidates who differentiate throughout, only •[1] is available.
2. For candidates who 'integrate the denominator' without attempting to write in integrable form award 0/5.
3. •[2] may be awarded for the appearance of $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$ in the line of working where the candidate first attempts to integrate. See Candidate F.
4. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket or use another invalid approach no further marks are available.
5. For •[2] to be awarded the integrand must contain a non-integer power.
6. Do not penalise the inclusion of '$+c$'.
7. •[4] and •[5] are not available to candidates who substitute into the original function.
8. The integral obtained must contain a non-integer power for •[5] to be available.
9. •[5] is only available to candidates who deal with the coefficient of x at the •[3] stage. See Candidate A.

Commonly Observed Responses:

Candidate A		Candidate B	
$(2x+9)^{-\frac{2}{3}}$	•[1] ✓	$(2x+9)^{\frac{2}{3}}$	•[1] ✗
$\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$	•[2] ✓ •[3] ^	$\dfrac{(2x+9)^{\frac{5}{3}}}{\frac{5}{3}}\times\dfrac{1}{2}$	•[2] $\boxed{✓\ 1}$ •[3] ✓
$3(2(9)+9)^{\frac{1}{3}}-3(2(-4)+9)^{\frac{1}{3}}$	•[4] $\boxed{✓\ 1}$	$\dfrac{3}{10}(2(9)+9)^{\frac{5}{3}}-\dfrac{3}{10}(2(-4)+9)^{\frac{5}{3}}$	•[4] $\boxed{✓\ 1}$
6	•[5] $\boxed{✓\ 2}$ see note 9	$\dfrac{363}{5}$	•[5] $\boxed{✓\ 1}$

Commonly Observed Responses:	
Candidate C $(2x+9)^{-\frac{2}{3}}$ •¹ ✓ $-\frac{5}{3}(2x+9)^{-\frac{5}{3}}\times\frac{1}{2}$ •² ✗ •³ ✓ $-\frac{5}{6}(2(9)+9)^{-\frac{5}{3}}-\left(-\frac{5}{6}(2(-4)+9)^{-\frac{5}{3}}\right)$ •⁴ $\boxed{\checkmark\ 1}$ $\frac{605}{729}$ •⁵ $\boxed{\checkmark\ 1}$	**Candidate D** $(2x+9)^{-\frac{2}{3}}$ •¹ ✓ $\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\times 2$ •² ✓ •³ ✗ $6(2(9)+9)^{\frac{1}{3}}-6(2(-4)+9)^{\frac{1}{3}}$ •⁴ $\boxed{\checkmark\ 1}$ 12 •⁵ $\boxed{\checkmark\ 1}$
Candidate E $(2x+9)^{-\frac{3}{2}}$ •¹ ✗ $\frac{(2x+9)^{-\frac{1}{2}}}{-\frac{1}{2}}\times\frac{1}{2}$ •² $\boxed{\checkmark\ 1}$ •³ ✓ $\left(-(2(9)+9)^{-\frac{1}{2}}\right)-\left(-(2(-4)+9)^{-\frac{1}{2}}\right)$ •⁴ $\boxed{\checkmark\ 1}$ $-\frac{1}{\sqrt{27}}+1$ •⁵ $\boxed{\checkmark\ 1}$	**Candidate F** $1\times(2x+9)^{-\frac{2}{3}}$ •¹ ✓ $x\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\times\frac{1}{2}$ •² ✓ •³ •⁴ and •⁵ are not available

Question	Generic scheme	Illustrative scheme	Max mark
15.	•¹ root at $x = -4$ identifiable from graph •² stationary point touching x-axis when $x = 2$ identifiable from graph •³ stationary point when $x = -2$ identifiable from graph •⁴ identify orientation of the cubic curve and $f'(0) > 0$ identifiable from graph	•¹ •² •³ •⁴ 	**4**

Notes:

1. For a diagram which does not show a cubic curve award 0/4.
2. For candidates who identify the roots of the cubic at '$x = -4, -2$ and 2' or at '$x = -2, 2$ and 4' •⁴ is unavailable.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

H

National
Qualifications
2018

2018 Mathematics

Higher - Paper 2

Finalised Marking Instructions

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2+5x+7=9x+4$$

$$x-4x+3=0$$

This is a transcription error and so the mark is not awarded.

$$x=1$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2+5x+7=9x+4$$

$$x-4x+3=0$$

$$(x-3)(x-1)=0$$

$$x=1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3 + 2x^2 + 3x + 2\right)\left(2x + 1\right)$ written as

  $\left(x^3 + 2x^2 + 3x + 2\right) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•¹ state an integral to represent the shaded area •² integrate •³ substitute limits •⁴ evaluate integral	•¹ $\int_{-1}^{3}\left(3+2x-x^2\right)dx$ •² $3x+\frac{2x^2}{2}-\frac{x^3}{3}$ •³ $\left(3\times3+\frac{2\times3^2}{2}-\frac{3^3}{3}\right)-\left(3\times(-1)+\frac{2\times(-1)^2}{2}-\frac{(-1)^3}{3}\right)$ •⁴ $\frac{32}{3}$ (units²)	4

Notes:

1. •¹ is not available to candidates who omit 'dx'.
2. Limits must appear at the •¹ stage for •¹ to be awarded.
3. Where a candidate differentiates one or more terms at •², then •³ and •⁴ are unavailable.
4. Candidates who substitute limits without integrating, do not gain •³ or •⁴.
5. Do not penalise the inclusion of '$+c$'.
6. Do not penalise the continued appearance of the integral sign after •¹.
7. If •⁴ is only given as a decimal then it must be given correct to 1 decimal place.

Commonly Observed Responses:

Candidate A		Candidate B	
$\int_{-1}^{3} 3+2x-x^2$	•¹ ✗	$\int\left(3+2x-x^2\right)dx$	•¹ ✗
$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	•² ✓	$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	•² ✓
	•³ ^	$=9-\left(-\frac{5}{3}\right)$	•³ ✓
$=\frac{32}{3}$	•⁴ ✓ 1	$=\frac{32}{3}$	•⁴ ✓

Commonly Observed Responses:	
Candidate C $\int\left(3+2x-x^2\right)dx$ •1 ✗ $=3x+\frac{2x^2}{2}-\frac{x^3}{3}$ •2 ✓ $=\left(3\times3+\frac{2\times3^2}{2}-\frac{3^3}{3}\right)$ $-\left(3\times(-1)+\frac{2\times(-1)^2}{2}-\frac{(-1)^3}{3}\right)$ •3 ✓ $=\frac{32}{3}$ •4 ✓	**Candidate D** $\int_3^{-1}\left(3+2x-x^2\right)dx$ •1 ✓ … •2 ✓•3 ✓ $=-\frac{32}{3}$, hence area is $\frac{32}{3}$ •4 ✓ However $-\frac{32}{3}=\frac{32}{3}$ does not gain •4.

Question		Generic scheme	Illustrative scheme	Max mark
2.	(a)	•[1] find $\mathbf{u.v}$	•[1] 24	1

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•[2] find $\lvert\mathbf{u}\rvert$ •[3] find $\lvert\mathbf{v}\rvert$ •[4] apply scalar product •[5] calculate angle	•[2] $\sqrt{26}$ •[3] $\sqrt{138}$ •[4] $\cos\theta° = \frac{24}{\sqrt{26}\sqrt{138}}$ •[5] $66\cdot38...°$ or $1\cdot16...$radians	4

Notes:

1. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{-1^2+4^2-3^2}=\sqrt{26}$ or $\sqrt{-1^2+4^2+-3^2}=\sqrt{26}$, •[2] is awarded.
2. •[4] is not available to candidates who simply state the formula $\cos\theta° = \frac{\mathbf{u.v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$.
3. Accept answers which round to $66°$ or $1\cdot2$ radians (or 73·8 gradians).
4. Do not penalise the omission or incorrect use of units.
5. •[5] is only available for a single angle.
6. For a correct answer with no working award 0/4.

Commonly Observed Responses:

Candidate A		
$\lvert\mathbf{u}\rvert=\sqrt{26}$	•[2] ✓	
$\lvert\mathbf{v}\rvert=\sqrt{138}$	•[3] ✓	
$\frac{24}{\sqrt{26}\sqrt{138}}$	•[4] ^	
$\theta = 66\cdot38\ldots°$	•[5] $\boxed{✓\ 1}$	

Question	Generic scheme	Illustrative scheme	Max mark
3.	•¹ differentiate	•¹ $3x^2 - 7$	**3**
	•² evaluate derivative at $x = 2$	•² 5	
	•³ interpret result	•³ $(f \text{ is})$ increasing	

Notes:

1. •² and •³ are only available as a consequence of working with a derivative.
2. Accept $f'(2) > 0$ for •².
3. $f'(x) > 0$ with no evidence of evaluating the derivative at $x = 2$ does not gain •² or •³. See candidate B.
4. Do not penalise candidates who use y in place of $f(x)$.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$3x^2 - 7$	•¹ ✓	$3x^2 - 7$	•¹ ✓
$\begin{array}{c\|c} x & 2 \\ \hline f'(x) & + \end{array}$	•² ✓	$f'(x) > 0$	•² ^
increasing	•³ ✓	f is increasing	•³ ^

Question	Generic scheme	Illustrative scheme	Max mark
4.	**Method 1**	**Method 1**	3
	•¹ identify common factor	•¹ $-3\left(x^2+2x\ldots\right.$ stated or implied by •²	
	•² complete the square	•² $-3(x+1)^2\ldots$	
	•³ process for c	•³ $-3(x+1)^2+10$	
	Method 2	**Method 2**	
	•¹ expand completed square form	•¹ $ax^2+2abx+ab^2+c$	
	•² equate coefficients	•² $a=-3$, $2ab=-6$ $ab^2+c=7$	
	•³ process for b and c and write in required form	•³ $-3(x+1)^2+10$	

Notes:

1. $-3(x+1)^2+10$ with no working gains •¹ and •² only; however, see Candidate E.
2. •³ is only available for a calculation involving both multiplication and addition of integers.

Commonly Observed Responses:

Candidate A

$-3\left(x^2+2\right)+7$ exception in General marking principle (h)

$-3\left(\left(x+1\right)^2-1\right)+7$ •¹ ✓•² ✓

$-3(x+1)^2+10$ •³ ✓

Candidate B

$-3\left(\left(x^2-6x\right)+7\right)$ •¹ ✗

$-3\left(\left(x-3\right)^2-9\right)+7$ •² ✓ 1

$-3(x-3)^2+34$ •³ ✓ 1

Candidate C

$a(x+b)^2+c=ax^2+2abx+ab^2+c$ •¹ ✓

$a=-3,\ \ 2ab=-6,\ \ ab^2+c=7$ •² ✓

$b=1,\ \ c=10$ •³ ✓

•³ is awarded as all working relates to completed square form

Candidate D

$ax^2+2abx+ab^2+c$ •¹ ✓

$a=-3,\ \ 2ab=-6,\ \ ab^2+c=7$ •² ✓

$b=1,\ \ c=10$ •³ ✗

•³ is lost as no reference is made to completed square form

Commonly Observed Responses:	
Candidate E $-3(x+1)^2+10$ Check: $=-3(x^2+2x+1)+10$ $=-3x^2-6x-3+10$ $=-3x^2-6x+7$ Award 3/3	**Candidate F** $-3x^2-6x+7$ $=-3(x+1)^2-1+7$ $\bullet^1$ ✓ $\bullet^2$ ✓ $=-3(x+1)^2+6$ $\bullet^3$ ✗
Candidate G $-3x^2-6x+7$ $=x^2+2x-\frac{7}{3}$ $\bullet^1$ ✗ $=(x+1)^2-\frac{10}{3}$ $\bullet^2$ ✗ $=-3(x+1)^2+10$ $\bullet^3$ ✗	

Question	Generic scheme	Illustrative scheme	Max mark
5. (a)	$\bullet^1$ find the midpoint of PQ $\bullet^2$ calculate m_{PQ} and state perp. gradient $\bullet^3$ find equation of L_1 in a simplified form	$\bullet^1$ $(6,1)$ $\bullet^2$ $-1 \Rightarrow m_{\text{perp}} = 1$ $\bullet^3$ $y = x - 5$	**3**
Notes:			
1. $\bullet^3$ is only available as a consequence of using a perpendicular gradient and a midpoint. 2. The gradient of the perpendicular bisector must appear in simplified form at $\bullet^2$ or $\bullet^3$ stage for $\bullet^3$ to be awarded. 3. At $\bullet^3$, accept $x - y - 5 = 0$, $y - x = -5$ or any other rearrangement of the equation where the constant terms have been simplified.			
Commonly Observed Responses:			
(b)	$\bullet^4$ determine y coordinate $\bullet^5$ state x coordinate	$\bullet^4$ 5 $\bullet^5$ 10	**2**
Notes:			
Commonly Observed Responses:			
(c)	$\bullet^6$ calculate radius of the circle $\bullet^7$ state equation of the circle	$\bullet^6$ $\sqrt{50}$ stated or implied by $\bullet^7$ $\bullet^7$ $(x-10)^2 + (y-5)^2 = 50$	**2**
Notes:			
4. Where candidates have calculated the coordinates of C incorrectly, $\bullet^6$ and $\bullet^7$ are available for using either PC or QC for the radius. 5. Where incorrect coordinates for C appear without working, only $\bullet^7$ is available. 6. Do not accept $\left(\sqrt{50}\right)^2$ for $\bullet^7$.			
Commonly Observed Responses:			

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)	(i)	•¹ start composite process •² substitute into expression	•¹ $f(2x)$ •² $3+\cos 2x$	**2**
		(ii)	•³ state second composite	•³ $2(3+\cos x)$	**1**

Notes:

1. For $3+\cos 2x$ without working, award both •¹ and •².
2. Candidates who interpret the composite function as either $g(x)\times f(x)$ or $g(x)+f(x)$ do not gain any marks.

Commonly Observed Responses:

Candidate A - interpret $f(g(x))$ as $g(f(x))$		**Candidate B - interpret $f(g(x))$ as $g(f(x))$**	
(i) $2(3+\cos x)$	•¹ ✘ •² $\boxed{✓\ 1}$	(i) $f(2x)=2(3+\cos x)$	•¹ ✓ •² ✘
(ii) $3+\cos 2x$	•³ $\boxed{✓\ 1}$	(ii) $3+\cos(2x)$	•³ $\boxed{✓\ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
6. (b)	•4 equate expressions from (a)	•4 $3+\cos 2x = 2(3+\cos x)$	**6**
	•5 substitute for $\cos 2x$ in equation	•5 $3+2\cos^2 x - 1 = 2(3+\cos x)$	
	•6 arrange in standard quadratic form	•6 $2\cos^2 x - 2\cos x - 4 = 0$	
	•7 factorise	•7 $2(\cos x - 2)(\cos x + 1)$	
	•8 solve for $\cos x$	•8 •8 $\cos x = 2$ •9 $\cos x = -1$	
	•9 solve for x	•9 $\cancel{\cos x = 2}$ or eg 'no solution' $x = \pi$	

Notes:

3. Do not penalise absence of common factor at •7.
4. •5 cannot be awarded until the equation reduces to a quadratic in $\cos x$.
5. Substituting $2\cos^2 A - 1$ or $2\cos^2 \alpha - 1$ at •5 stage should be treated as bad form provided the equation is written in terms of x at •6 stage. Otherwise, •5 is not available.
6. '$=0$' must appear by •7 stage for •6 to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at •6 stage for •6 to be awarded.
7. For candidate who do not arrange in standard quadratic form, eg $-2\cos x + 2\cos^2 x - 4 = 0$ •6 is only available if •7 has been awarded.
8. •7 •8 and •9 are only available as a consequence of solving a quadratic with distinct real roots.
9. •7 •8 and •9 are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$.
10. •9 is not available to candidates who work in degrees and do not convert their solution(s) into radian measure.
11. Answers written as decimals should be rounded to no fewer than 2 significant figures.
12. •9 is not available for any solution containing angles outwith the interval $0 \le x < 2\pi$.

Commonly Observed Responses:	
Candidate C **Quadratic expressed in terms of c or x.** $3+\cos 2x=2(3+\cos x)$ •4 ✓ $3+2\cos^2 x-1=2(3+\cos x)$ •5 ✓ $2\cos^2 x-2\cos x-4=0$ •6 ✓ $2c^2-2c-4=0$ $2(c-2)(c+1)=0$ •7 ✓ $c=2$, $c=-1$ •8 ✘ no solution, $x=\pi$ •9 ✓ However, •4 ✓ •5 ✓ •6 ✓ $2(c-2)(c+1)=0$ •7 ✓ $\cos x=2$ $\cos x=-1$ •8 ✓ Solution stated in terms of $\cos x$ explicitly	**Candidate D** $3+\cos 2x=2(3+\cos x)$ •4 ✓ $3+2\cos^2 x-1=2(3+\cos x)$ •5 ✓ $2\cos^2 x-2\cos x=4$ •6 ^ $\cos^2 x-\cos x=2$ $\cos x(\cos x-1)=2$ •7 $\boxed{✓ 2}$ $\cos x=2,\ \cos x-1=2$ $\cos x=2,\ \cos x=3$ •8 ✘ no solutions •9 ✘ see note 9
Candidate E - reading $\cos 2x$ as $\cos^2 x$ $3+\cos^2 x=2(3+\cos x)$ •4 ✘ •5 ^ - no substitution required $\cos^2 x-2\cos x-3=0$ •6 $\boxed{✓ 1}$ $(\cos x-3)(\cos x+1)$ •7 $\boxed{✓ 1}$ $\cos x=3,\ \ \cos x=-1$ •8 $\boxed{✓ 1}$ no solution, $x=\pi$ •9 $\boxed{✓ 1}$	**Candidate F - using quadratic formula** 4 ✓ •5 ✓ $2\cos^2 x-2\cos x-4=0$ •6 ✓ $\cos x=\dfrac{2\pm\sqrt{36}}{4}$ or $\cos x=\dfrac{1\pm\sqrt{9}}{2}$ •7 ✓

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)	(i)	•¹ use ‘2’ in synthetic division or in evaluation of cubic •² complete division/evaluation and interpret result	•¹ $\begin{array}{r\|rrrr} 2 & 2 & -3 & -3 & 2 \\ & & & & \\ \hline & 2 & & & \end{array}$ or $2\times(2)^3-3(2)^2-3\times(2)+2$ •² $\begin{array}{r\|rrrr} 2 & 2 & -3 & -3 & 2 \\ & & 4 & 2 & -2 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$ Remainder $=0$ $\therefore (x-2)$ is a factor or $f(2)=0$ $\therefore (x-2)$ is a factor	**2**
		(ii)	•³ state quadratic factor •⁴ complete factorisation	•³ $2x^2+x-1$ •⁴ $(x-2)(2x-1)(x+1)$ stated explicitly	**2**

Notes:

1. Communication at •² must be consistent with working at that stage i.e. a candidate’s working must arrive legitimately at 0 before •² can be awarded.
2. Accept any of the following for •² :
   - ‘ $f(2)=0$ so $(x-2)$ is a factor’
   - ‘since remainder = 0, it is a factor’
   - the 0 from any method linked to the word ‘factor’ by e.g. ‘so’, ‘hence’, ‘$\therefore$’, ‘$\rightarrow$’, ‘$\Rightarrow$’
3. Do not accept any of the following for •²:
   - double underlining the zero or boxing the zero without comment
   - ‘ $x=-2$ is a factor’, ‘$(x+2)$ is a factor’, ‘$(x+2)$ is a root’, ‘ $x=2$ is a root’, ‘$(x-2)$ is a root’, ‘ $x=-2$ is a root’
   - the word ‘factor’ only, with no link.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
7.	(b)	•⁵ demonstrate result	•⁵ $u_6=a(2a-3)-1=2a^2-3a-1$ leading to $u_7=a(2a^2-3a-1)-1$ $=2a^3-3a^2-a-1$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(c)	(i)	•6 equate u_5 and u_7 and arrange in standard form •7 solve cubic •8 discard invalid solutions for a	•6 $2a^3-3a^2-3a+2=0$ •7 $a=2,\ a=\frac{1}{2},\ a=-1$ •8 $a=\frac{1}{2}$	3
		(ii)	•9 calculate limit	•9 -2	1

Notes:

4. Where •6 has been awarded, •7 is available for solutions in terms of x appearing in a(ii). However, see Candidates B and C. BEWARE: Candidates who make a second attempt at factorising the cubic obtained in c(i) and do so incorrectly cannot be awarded mark 7 for solutions appearing in a(ii).
5. •8 is only available as a result of a valid strategy at •7.
6. $x=\frac{1}{2}$ does not gain •8.
7. For candidates who do not state the cubic equation at •6, and adopt a guess and check approach, using solutions for x found in a(ii), may gain 3/3. See Candidate D.

Commonly Observed Responses:

Candidate A $2a^3-3a^2-3a+2=0$ •6 ✓ $x=2,\ x=\frac{1}{2},\ x=-1$ in a(ii) •7 ✓ •8 ^	**Candidate B - missing '$=0$' from equation** $2a^3-3a^2-3a+2$ •6 ^ $\cancel{x=2},\ x=\frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 $\boxed{✓ 1}$ $a=\frac{1}{2}$ •8 $\boxed{✓ 1}$
Candidate C - missing '$=0$' from equation $2a^3-3a^2-3a+2$ •6 ^ $\cancel{x=2},\ x=\frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 ^ $\frac{1}{2}$ •8 ^ No clear link between a and x.	**Candidate D - $x=-1,\ x=\frac{1}{2}$ and $x=2$ identified in a(ii)** $u_5=2\left(\frac{1}{2}\right)-3=-2$ •6 ✓ $u_7=2\left(\frac{1}{2}\right)^3-3\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)-1=-2$ •7 ✓ $a=\frac{1}{2}$ because $-1<a<1$ •8 ✓

Question		Generic scheme	Illustrative scheme	Max mark
8.	(a)	•¹ use compound angle formula •² compare coefficients •³ process for k •⁴ process for a and express in required form	•¹ $k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ stated explicitly •² $k\cos a^\circ = 2$ and $k\sin a^\circ = -1$ stated explicitly •³ $k = \sqrt{5}$ •⁴ $\sqrt{5}\cos(x - 333\cdot4\ldots)^\circ$	**4**

Notes:

1. Accept $k(\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ)$ for $\bullet^1$. Treat $k\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
2. Do not penalise the omission of degree signs.
3. $\sqrt{5}\cos x^\circ\cos a^\circ + \sqrt{5}\sin x^\circ\sin a^\circ$ or $\sqrt{5}(\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x^\circ = 2$, $k\sin x^\circ = -1$, however $\bullet^4$ may still be gained.
5. $\bullet^3$ is only available for a single value of k, $k > 0$.
6. $\bullet^4$ is not available for a value of a given in radians.
7. Accept any value of a which rounds to 333°
8. Candidates may use any form of the wave function for $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the wave is interpreted in the form $k\cos(x - a)^\circ$.
9. Evidence for $\bullet^4$ may not appear until part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A	**Candidate B**	**Candidate C**
•¹ ^ $\sqrt{5}\cos a^\circ = 2$ $\sqrt{5}\sin a^\circ = -1$ •² ✓ •³ ✓ $\tan a^\circ = -\frac{1}{2}$ $a = 333\cdot4$ $\sqrt{5}\cos(x - 333\cdot4)^\circ$ •⁴ ✓	$k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ •¹ ✓ $\cos a^\circ = 2$ $\sin a^\circ = -1$ •² ✗ $\tan a^\circ = -\frac{1}{2}$ $a = 333\cdot4$ Not consistent with equations at •². $\sqrt{5}\cos(x - 333\cdot4)^\circ$ •³ ✓ •⁴ ✗	$\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ$ •¹ ✗ $\cos a^\circ = 2$ $\sin a^\circ = -1$ •² ✗ $k = \sqrt{5}$ •³ ✓ $\tan a^\circ = -\frac{1}{2}$ $a = 333\cdot4$ $\sqrt{5}\cos(x - 333\cdot4)^\circ$ •⁴ ✗

Responses with the correct expansion of $k\cos(x-a)°$ but errors for either $\bullet^2$ or $\bullet^3$:

Candidate D	Candidate E	Candidate F
$k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$ ✓ $k\cos a° = 2$ $k\sin a° = -1$ $\bullet^2$ ✓ $\tan a° = -2$ $\bullet^3$ ^ $\bullet^4$ ✗ $a = 296\cdot6$	$k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$ ✓ $k\cos a° = -1$ $k\sin a° = 2$ $\bullet^2$ ✗ $\tan a° = -2$ $a = 116\cdot6$ $\sqrt{5}\cos(x-116\cdot6)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$	$k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$ ✓ $k\cos a° = 2$ $k\sin a° = 1$ $\bullet^2$ ✗ $\tan a° = \frac{1}{2}$ $a = 26\cdot6$ $\sqrt{5}\cos(x-26\cdot6)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$

Commonly Observed Responses:

Responses with the incorrect labelling, $k(\cos A\cos B + \sin A\sin B)$ from the formula list:

Candidate G	Candidate H	Candidate I
$k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos a° = 2$ $k\sin a° = -1$ $\bullet^2$ ✓ $\tan a° = -\frac{1}{2}$ $a = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)°$ $\bullet^3$ ✓ $\bullet^4$ ✓	$k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos x° = 2$ $k\sin x° = -1$ $\bullet^2$ ✗ $\tan x° = -\frac{1}{2}$ $x = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$	$k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos B° = 2$ $k\sin B° = -1$ $\bullet^2$ ✗ $\tan B° = -\frac{1}{2}$ $B = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)	(i)	•5 state minimum value	•5 $-3\sqrt{5}$ or $-\sqrt{45}$	1
		(ii)	**Method 1** •6 start to solve •7 state value of x **Method 2** •6 start to solve •7 state value of x	**Method 1** •6 $x-333\cdot4=180$ leading to $x=513\cdot4$ •7 $x=153\cdot4\ldots$ **Method 2** •6 $x-333\cdot4=-180$ •7 $x=153\cdot4\ldots$	2

Notes:

10. •7 is only available for a single value of x.
11. •7 is only available in cases where $a<-180$ or $a>180$. See Candidate J

Commonly Observed Responses:

Candidate J - from $\sqrt{5}\cos(x-26\cdot6)^\circ$ $x-26\cdot6=180$ $x=206\cdot6$ •6 ✓ 1 •7 ✓ 2 Similarly for $\sqrt{5}\cos(x-116\cdot6)^\circ$	**Candidate K - from 'minimum' of eg $-\sqrt{5}$** $3\sqrt{5}\cos(x-333\cdot4)^\circ=-\sqrt{5}$ $\cos(x-333\cdot4)^\circ=-\frac{1}{3}$ $x-333\cdot4=109\cdot5,\ 250\cdot5$ $x=442\cdot9,\ 583\cdot9$ •6 ✓ 1 $x=82\cdot9,\ 223\cdot9$ •7 ✗

Question	Generic scheme	Illustrative scheme	Max mark
9.	•¹ express P in differentiable form •² differentiate •³ equate expression for derivative to 0 •⁴ process for x •⁵ verify nature •⁶ evaluate P	•¹ $2x+128x^{-1}$ •² $2-\frac{128}{x^2}$ •³ $2-\frac{128}{x^2}=0$ •⁴ 8 •⁵ table of signs for a derivative (see next page) $\therefore$ minimum or $P''(8)=\frac{1}{2}>0$ $\therefore$ minimum •⁶ $P=32$ or min value $=32$	6

Notes:

1. For a numerical approach award 0/6.
2. For candidates who integrate any term at the •² stage, only •³ is available on follow through for setting their 'derivative' to 0.
3. •⁴, •⁵ and •⁶ are only available for working with a derivative which contains an index ≤ -2.
4. At •² accept $2-128x^{-2}$.
5. Ignore the appearance of -8 at •⁴.
6. $\sqrt{\frac{128}{2}}$ must be simplified at •⁴ or •⁵ for •⁴ to be awarded.
7. •⁵ is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 8.
8. •⁶ is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at •⁵.
9. •⁵ and •⁶ are not available to candidates who state that the minimum exists at a negative value of x.

Commonly Observed Responses:

Candidate A - differentiating over more than one line		Candidate B - differentiating over more than one line	
	•¹ ^	$P(x)=2x+128x^{-1}$	•¹ ✓
$P'(x)=2+128x^{-1}$		$P'(x)=2+128x^{-1}$	
$P'(x)=2-128x^{-2}$	•² ✗	$P'(x)=2-128x^{-2}$	•² ✗
$2-128x^{-2}=0$	•³ $\boxed{✓\ 1}$	$2-128x^{-2}=0$	•³ $\boxed{✓\ 1}$

Table of signs for a derivative

Accept:

x	-8^-	-8	-8^+
$P'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	8^-	8	8^+
$P'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

x	$\rightarrow$	-8	$\rightarrow$
$P'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	$\rightarrow$	8	$\rightarrow$
$P'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Arrows are taken to mean 'in the neighbourhood of'

Here, for exemplification, tables of signs considering both roots separately have been displayed. However, in this question, it was only expected that candidates would consider the positive root for $\bullet^5$. Do not penalise the consideration of the negative root.

x	a	-8	b	c	8	d
$P'(x)$	$+$	0	$-$	$-$	0	$+$
Shape or Slope	/	—	\	\	—	/

Where: $a < -8$, $-8 < b < 0$, $0 < c < 8$, $d > 8$

Do not Accept:

x	a	-8	b	8	c
$P'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-8 < b < 8$' is not acceptable.

$P'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-8 \rightarrow 8$' is not acceptable.

General Comments:

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $P'(x)$ in the table is an acceptable alternative to writing '$+$' or '$-$' signs. Values must be checked for accuracy.
  - The only acceptable variations of $P'(x)$ are: P', $\frac{dP}{dx}$ and $2 - \frac{128}{x^2}$.

Question	Generic scheme	Illustrative scheme	Max mark
10.	•[1] use the discriminant •[2] identify roots of quadratic expression •[3] apply condition •[4] state range with justification	•[1] $(m-3)^2-4\times1\times m$ •[2] 1, 9 •[3] $(m-3)^2-4\times1\times m>0$ •[4] $m<1,\ m>9$ with eg sketch or table of signs	4

Notes:

1. If candidates have the condition '$\text{discriminant}<0$', '$\text{discriminant}\leq0$' or '$\text{discriminant}\geq0$', then •[3] is lost but •[4] is available.
2. Ignore the appearance of $b^2-4ac=0$ where the correct condition has subsequently been applied.
3. For candidates who have identified expressions for a, b, and c and then state $b^2-4ac>0$ award •[3]. See Candidate A.
4. For the appearance of x in any expression for •[1], award 0/4.

Commonly Observed Responses:

Candidate A	
$(m-3)^2-4\times1\times m$	•[1] ✓
$m^2-10m+9=0$	
$m=1,\ m=9$	•[2] ✓
$b^2-4ac>0$	•[3] ✓
$m<1,\ m>9$	•[4] ^
Expressions for a, b, and c implied at •[1]	

Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•[1] substitute for P and t •[2] arrange equation in the form $A=e^{kt}$ •[3] simplify •[4] solve for k	•[1] $50=100\left(1-e^{3k}\right)$ •[2] $0\cdot5=e^{3k}$ or $-0\cdot5=-e^{3k}$ •[3] $\ln 0\cdot5=3k$ •[4] $k=-0\cdot231$	4

Notes:

1. •[2] may be assumed by •[3].
2. Any base may be used at •[3] stage. See Candidate D.
3. Accept any answer which rounds to $-0\cdot2$.
4. •[3] must be consistent with the equation of the form $A=e^{kt}$ at its first appearance.
5. For candidates whose working would (or should) arrive at $\log(\text{negative})$ •[4] is not available.
6. Where candidates use a 'rule' masquerading as a law of logarithms, •[3] and •[4] is not available.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$50=100\left(1-e^{3k}\right)$	•[1] ✓	$0\cdot5=100\left(1-e^{3k}\right)$	•[1] ✗
$0\cdot5=-e^{3k}$	•[2] ✗	$0\cdot995=e^{3k}$	•[2] $\boxed{✓\ 1}$
$\ln(0\cdot5)=3k$	•[3] ✗	$\ln(0\cdot995)=3k$	•[3] $\boxed{✓\ 1}$
$k=-0\cdot231$	•[4] ✗	$k=-0\cdot0017\ldots$	•[4] $\boxed{✓\ 1}$
68·5	•[5] $\boxed{✓\ 1}$	$P=0\cdot8319\ldots$	•[5] $\boxed{✓\ 1}$
31·5% still queueing	•[6] $\boxed{✓\ 1}$	99·2% still queuing	•[6] $\boxed{✓\ 1}$
Candidate C		**Candidate D**	
$50=100\left(1-e^{3k}\right)$	•[1] ✓	$50=100\left(1-e^{3k}\right)$	•[1] ✓
$-0\cdot5=-e^{3k}$	•[2] ✓	$0\cdot5=e^{3k}$	•[2] ✓
$\ln(-0\cdot5)=\ln\left(-e^{3k}\right)$	•[3] ✗	$\log_{10}(0\cdot5)=3k\log_{10}e$	•[3] ✓
$k=-0\cdot231$	•[4] ✗	$k=-0\cdot231$	•[4] ✓
68·5	•[5] $\boxed{✓\ 1}$		
31·5% still queueing	•[6] $\boxed{✓\ 1}$		

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•[5] evaluate P for $t=5$ •[6] interpret result	•[5] 68·5 •[6] 31·5% still queueing	2

Notes:

7. •[5] and •[6] are not available where $k\geq0$.
8. •[6] is only available where the value of P in •[5] was obtained by substituting into an exponential expression.

Commonly Observed Responses:

Candidate D - $k=-0\cdot2$		
63·2	•[5] ✓	
36·8% still queueing	•[6] ✓	

Question	Generic scheme	Illustrative scheme	Max mark
12. (a) (i)	•¹ write down coordinates of centre	•¹ $(13,-4)$	**1**
(ii)	•² substitute coordinates and process for c	•² $13^2+(-4)^2+14\times13-22\times(-4)\ldots$ leading to $c=-455$	**1**
Notes:			
1. Accept $x=13$, $y=-4$ for •¹. 2. Do not accept $g=13$, $f=-4$ or 13, −4 for •¹. 3. For those who substitute into $r=\sqrt{g^2+f^2-c}$, working to find r must be shown for •² to be awarded.			
Commonly Observed Responses:			
(b) (i)	•³ calculate two key distances •⁴ state ratio	•³ two from $r_2=25$, $r_1=10$ and $r_2-r_1=15$ •⁴ $3:2$ or $2:3$	**2**
(ii)	•⁵ identify centre of C_2 •⁶ state coordinates of P	•⁵ $(-7,11)$ or $\begin{pmatrix}-7\\11\end{pmatrix}$ •⁶ $(5,2)$	**2**
Notes:			
4. The ratio must be consistent with the working for r_2-r_1 5. Evidence for •³ may appear on a sketch. 6. For $3:2$ or $2:3$ with no working, award 0/2. 7. At •⁶, the ratio used to identify the coordinates of P must be consistent with the sizes of the circles in the original diagram for •⁶ to be available.			
Commonly Observed Responses:			
(c)	•⁷ state equation	•⁷ $(x-5)^2+(y-2)^2=1600$ or $x^2+y^2-10x-4y-1571=0$	**1**
Notes:			
Commonly Observed Responses:			

[END OF MARKING INSTRUCTIONS]