H

National
Qualifications
RESOURCE

2021 Mathematics
Paper 2

Higher

Finalised Marking Instructions

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2+5x+7=9x+4$$

$$x-4x+3=0$$

This is a transcription error and so the mark is not awarded.

$$x=1$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2+5x+7=9x+4$$

$$x-4x+3=0$$

$$(x-3)(x-1)=0$$

$$x=1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	○5	○6
○5	$x = 2$	$x = -4$
○6	$y = 5$	$y = -7$

Horizontal: ○5 $x = 2$ and $x = -4$ Vertical: ○5 $x = 2$ and $y = 5$
○6 $y = 5$ and $y = -7$ ○6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0 \cdot 3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Section 1

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] differentiate •[2] evaluate y-coordinate •[3] evaluate $\frac{dy}{dx}$ •[4] state equation of tangent	•[1] $6x^2 - 16x$ •[2] -4 •[3] 6 •[4] eg $6x - y - 22 = 0$	**4**

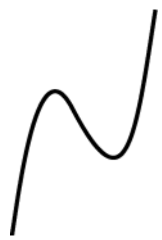
Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] write in integrable form •[2] start to integrate •[3] complete integration and simplify	•[1] $6(x+5)^{-\frac{3}{2}}$ •[2] $\dfrac{6(x+5)^{-\frac{1}{2}}}{-\frac{1}{2}}$ •[3] $-12(x+5)^{-\frac{1}{2}}+c$	3

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] start to differentiate •[2] complete differentiation •[3] evaluate rate of change	•[1] $\cos\left(2t+\frac{\pi}{6}\right)\ldots$ •[2] $\ldots\times 2$ •[3] $-0\cdot 206\ldots$	3

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•4 find gradient of AC •5 determine gradient of L_1 •6 determine equation of altitude	•4 $-\frac{2}{3}$ •5 $\frac{3}{2}$ •6 $3x-2y=7$	**3**
	(b)		•1 find midpoint of AB •2 find gradient of AB •3 find equation	•1 $(-1,1)$ •2 0 •3 m_{perp} undefined $\Rightarrow$ $x=-1$	**3**
	(c)		•7 state coordinates	•7 $x=-1$, $y=-5$	**1**

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•1 use compound angle formula •2 compare coefficients •3 process for k •4 process for a and express in required form	•1 $k\sin t^\circ \cos a^\circ + k\cos t^\circ \sin a^\circ$ stated explicitly •2 $k\sin a^\circ = 3$ and $k\cos a^\circ = 5$ stated explicitly •3 $k = \sqrt{34}$ •4 $\sqrt{34}\sin\left(t + 30\cdot96...\right)^\circ$	4
	(b)	(i)	•5 state minimum	•5 $-\sqrt{34}$	1
		(ii)	•6 state value of t	•6 $239\cdot0\ldots$	1

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] differentiate one term	•[1] $6\ldots$ or $\ldots - 3x^{\frac{1}{2}}$	**3**
			•[2] complete differentiation and equate to zero	•[2] $6 - 3x^{\frac{1}{2}} = 0$	
			•[3] solve for x	•[3] $x = 4$	
	(b)		•[4] express area as a definite integral	•[4] $\int_{\ldots}^{\ldots} \left(6x - 2x^{\frac{3}{2}}\right) dx$	**4**
			•[5] integrate	•[5] $3x^2 - \dfrac{2x^{\frac{5}{2}}}{\frac{5}{2}}$	
			•[6] substitute limits	•[6] $\left(3 \times 9^2 - \dfrac{4}{5} \times 9^{\frac{5}{2}}\right) - \left(3 \times 4^2 - \dfrac{4}{5} \times 4^{\frac{5}{2}}\right)$	
			•[7] evaluate area	•[7] $\dfrac{131}{5}$ or $26 \cdot 2$	

Question			Generic scheme	Illustrative scheme	Max mark
7.			•[1] identify roots •[2] interpret point of inflection •[3] complete curve	•[1] -1 and 3 •[2] turning point at $(3,0)$ •[3] correct shape, passing through $(-1,0)$ with a positive gradient 	**3**

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] solve for $\sin(3x-60)°$ •[2] find two solutions for $(3x-60)$ •[3] find corresponding values of x •[4] find remaining solution	•[1] $\sin(3x-60)° = -\frac{1}{2}$ •[2] eg 210, 330 •[3] 90, 130 •[4] 10	**4**

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] obtain expression for area in terms of r and h •[2] obtain an expression for h •[3] demonstrate result	•[1] $2\pi r^2 + 2\pi rh$ •[2] $h = \frac{450}{\pi r^2}$ •[3] $2\pi r^2 + 2\pi r\left(\frac{450}{\pi r^2}\right)$ leading to $A(r) = 2\pi r^2 + \frac{900}{r}$	**3**
	(b)		•[4] express A in differentiable form •[5] differentiate •[6] equate expression for derivative to 0 •[7] solve for r •[8] verify nature of stationary point •[9] interpret and communicate result	•[4] $2\pi r^2 + 900r^{-1}$ stated or implied by •[5] •[5] $4\pi r - 900r^{-2}$ •[6] $4\pi r - 900r^{-2} = 0$ •[7] $r = \sqrt[3]{\frac{225}{\pi}}$ •[8] table of signs for a derivative •[9] minimum when $r = \sqrt[3]{\frac{225}{\pi}}$ **OR** •[8] $A''(r) = 4\pi + \frac{1800}{r^3}$ •[9] $A''\left(\sqrt[3]{\frac{225}{\pi}}\right) > 0$ so minimum when $r = \sqrt[3]{\frac{225}{\pi}}$	**6**

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•[1] substitute for $\tan x$ •[2] simplify	•[1] $2\frac{\sin x}{\cos x}\cos^2 x$ **stated explicitly** •[2] $2\sin x\cos x$ $=\sin 2x$ **stated explicitly**	**2**
	(b)		•[3] link to (a) and know to integrate •[4] start integration •[5] complete integration •[6] state equation	•[3] $\int 3\sin 2x\,dx$ •[4] $-3\cos 2x\ldots$ •[5] $\ldots\times\frac{1}{2}+c$ •[6] $y=-\frac{3}{2}\cos 2x+\frac{9}{2}$	**4**

Section 2

Part A

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•[1] find $\overrightarrow{AB}$	•[1] $\begin{pmatrix} -5 \\ 4 \\ -7 \end{pmatrix}$	**2**
			•[2] find $\overrightarrow{AC}$	•[2] $\begin{pmatrix} 4 \\ -7 \\ -5 \end{pmatrix}$	
	(b)		•[3] evaluate $\overrightarrow{AB}.\overrightarrow{AC}$	•[3] -13	**4**
			•[4] evaluate $\lvert\overrightarrow{AB}\rvert$ and $\lvert\overrightarrow{AC}\rvert$	•[4] $\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{AC}\rvert = \sqrt{90}$	
			•[5] use scalar product	•[5] $\cos BAC = \dfrac{-13}{\sqrt{90}\sqrt{90}}$	
			•[6] calculate angle	•[6] $98\cdot30\ldots^\circ$ or $1\cdot715\ldots$ radians	

Question			Generic scheme	Illustrative scheme	Max mark
12.			•[1] interpret given information •[2] form an equation in u_k •[3] solve for u_k	•[1] $u_{k+1} - u_k = 1000$ stated or implied by •[2] •[2] $9u_k - 440 - u_k = 1000$ •[3] 180	3

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•¹ find $\overrightarrow{CF}$	•¹ $\begin{pmatrix}14\\6\\-5\end{pmatrix}$	**1**
	(b)		•² find $\overrightarrow{DF}$	•² $\begin{pmatrix}18\\4\\-2\end{pmatrix}$	**1**
	(c)		•³ use an appropriate relationship •⁴ find $\overrightarrow{QD}$	•³ for example $\overrightarrow{QD}=\overrightarrow{QF}+\overrightarrow{FD}$ •⁴ $\begin{pmatrix}-1\\1\\2\end{pmatrix}$	**2**

Part B

Question			Generic scheme	Illustrative scheme	Max mark
14.			•[1] state centre of circle •[2] find gradient of radius •[3] state gradient of tangent •[4] determine equation of tangent	•[1] $(5,-1)$ •[2] -3 •[3] $\frac{1}{3}$ •[4] for example $3y = x + 12$	**4**

Question			Generic scheme	Illustrative scheme	Max mark
15.			•[1] substitute $4-2x$ •[2] express in standard quadratic form •[3] find x-coordinates •[4] find y-coordinates	•[1] $x^2+(4-2x)^2-10x-8(4-2x)+1=0$ •[2] $5x^2-10x-15=0$ •[3] $x=-1,\ x=3$ •[4] $y=6,\ y=-2$	**4**

Question			Generic scheme	Illustrative scheme	Max mark
16.			**Method 1**	**Method 1**	5
			•1 state linear equation	•1 $\log_8 y = \frac{1}{3}x + 2$	
			•2 introduce logs	•2 $\log_8 y = \frac{1}{3}x\log_8 8 + 2\log_8 8$	
			•3 use laws of logs	•3 $\log_8 y = \log_8 8^{\frac{1}{3}x} + \log_8 8^2$	
			•4 use laws of logs	•4 $\log_8 y = \log_8 8^2.8^{\frac{1}{3}x}$	
			•5 state a and b	•5 $a = 64$, $b = 2$	
			Method 2	**Method 2**	
			•1 state linear equation	•1 $\log_8 y = \frac{1}{3}x + 2$	
			•2 convert to exponential form	•2 $y = 8^{\frac{1}{3}x+2}$	
			•3 use laws of indices	•3 $y = 8^{\frac{1}{3}x} \cdot 8^2$	
			•4 state a	•4 $a = 64$	
			•5 state b	•5 $b = 2$	
			Method 3	**Method 3** **The equations at •1, •2, •3 and •4 must be stated explicitly**	
			•1 introduce logs to $y = ab^x$	•1 $\log_8 y = \log_8 ab^x$	
			•2 use laws of logs	•2 $\log_8 y = \log_8 a + x\log_8 b$	
			•3 interpret intercept	•3 $2 = \log_8 a$	
			•4 interpret gradient	•4 $\frac{1}{3} = \log_8 b$	
			•5 state a and b	•5 $a = 64$, $b = 2$	

Question			Generic scheme	Illustrative scheme	Max mark
16.			**Method 4**	**Method 4**	
			•1 interpret point on log graph	•1 $x = 6$ **and** $\log_8 y = 4$	
			•2 convert from log to exponential form	•2 $x = 6$ **and** $y = 8^4$	
			•3 interpret point and convert	•3 $x = 0$ **and** $\log_8 y = 2$ $x = 0$ **and** $y = 8^2$	
			•4 substitute into $y = ab^x$ and evaluate a	•4 $8^2 = ab^0 \Rightarrow a = 64$	
			•5 substitute other point into $y = ab^x$ and evaluate b	•5 $8^4 = 64b^6 \Rightarrow b = 2$	

[END OF MARKING INSTRUCTIONS]