

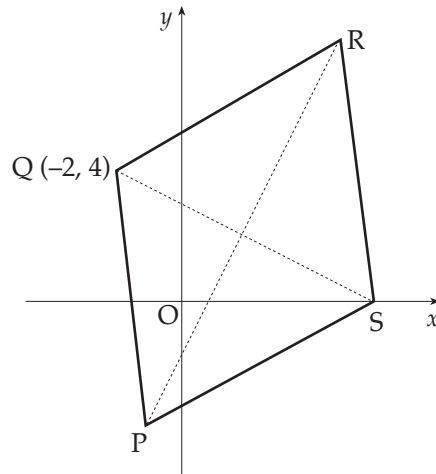
part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	6	1.1.10	CN	C	2002W q1
b)	2	1.1.8	CN	C	

The diagram shows a rhombus PQRS with its diagonals PR and QS.

PR has equation $y = 2x - 2$.

Q has coordinates $(-2, 4)$.

- (a) (i) Find the equation of the diagonal QS.
 (ii) Find the coordinates of T, the point of intersection of PR and QS.
- (b) R is the point $(5, 8)$. Write down the coordinates of P.



6

2

Give 1 mark for each •

Illustrations for awarding each •

a ans: $2y + x = 6$, $T(2, 2)$ 6 marks

- ¹ ss : know to & find gradient thr' 2 pts
- ² ss : know and use product of perp. grads
- ³ ic : state equ. of st line
- ⁴ pd : simplify
- ⁵ ss : know to solve sim equ. and start
- ⁶ pd : complete solving process

- ¹ $m_{PR} = 2$ **stated or implied by** •²
- ² use $m_1 m_2 = -1 \Rightarrow m_{QS} = -\frac{1}{2}$
- ³ $y - 4 = -\frac{1}{2}(x + 2)$
- ⁴ $2y + x = 6$ or equivalent
- ⁵ strategy for sim. equ eg substitute
- ⁶ $T(2, 2)$

b ans: $P(-1, -4)$ 2 marks

- ⁷ ss : choose strategy eg use R and T
- ⁸ pd : complete chosen strategy

- OR**
- ⁷ strategy eg $\vec{TP} = \vec{RT}$ •⁷ $x_P = -1$
 - ⁸ $P(-1, -4)$ •⁸ $y_P = -4$

Notes

- 1 •³ is only available to candidates who have made an attempt to find the perp. gradient
- 2 alt for •⁷: $\vec{QP} = \vec{RS}$ **and** $\vec{TS} = \vec{QT}$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	4	3.1.6. 3.1.7	CN	C	2002W q2

With reference to a suitable set of coordinate axes, A, B and C are the points $(-8, 10, 20)$, $(-2, 1, 8)$ and $(0, -2, 4)$ respectively.

Show that A, B and C are collinear and find the ratio AB : BC.

4

Give 1 mark for each •

Illustrations for awarding each •

ans: proof, 3:1

4 marks

- ¹ ic : interpret information as vector
- ² ss : know to find $\underline{a} = k\underline{b}$
- ³ ic : complete proof
- ⁴ ic : interpret results

$$\bullet^1 \vec{AB} = \begin{pmatrix} 6 \\ -9 \\ -12 \end{pmatrix} \text{ or } \vec{BC} = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix} \text{ or } \vec{AC} = \begin{pmatrix} 8 \\ -12 \\ -16 \end{pmatrix}$$

$$\bullet^2 \text{ 2nd vector } \textit{and} \textit{ e.g. } \vec{AB} = 3\vec{BC} \left(\text{or } \begin{pmatrix} 6 \\ -9 \\ -12 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix} \right)$$

- ³ e.g. \vec{AB}, \vec{BC} have common direction, B common pt., so A, B, C collinear

$$\bullet^4 \text{ AB:BC} = 3:1$$

Notes

- 1 Accept $\text{AB} \parallel \text{BC}$ in lieu of common direction.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	3	1.4.5	CR	C	2002W q3
b)	2	1.4.3	CR	C	

(a)	Calculate the limit as $n \rightarrow \infty$ of the sequence defined by $u_{n+1} = 0.9u_n + 10$, $u_0 = 1$.	3
(b)	Determine the least value of n for which u_n is greater than half of this limit and the corresponding value of u_n .	2

Give 1 mark for each •

Illustrations for awarding each •

a ans: 100 3 marks

- ¹ ic : state conditions for limit to exist
- ² ss : know and use a limit rule
- ³ pd : complete limit calculations

- ¹ $-1 < a < 1$ **stated explicitly**
- ² $l = 0.9l + 10$ or *equiv. strat*
- ³ $l = 100$

b ans: 100 2 marks

- ⁴ pd : calculate terms
- ⁵ pd : complete processing

- ⁴ 10.9, 19.8, 27.8
- ⁵ $u_7 = 52.65$

Notes

1 10.9, 19.8, 27.8, 35.5, 41.54, 47.39, 52.65

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	4	3.4.1	NC	C	2002W q4
b)	2	3.4..3	NC	B	

(a)	Write $\sqrt{3} \sin x^\circ + \cos x^\circ$ in the form $k \sin(x + \alpha)^\circ$ where $k > 0$ and $0 \leq \alpha < 360$.	4
(b)	Hence find the maximum value of $5 + \sqrt{3} \sin x^\circ + \cos x^\circ$ and determine the corresponding value of x in the interval $0 \leq x \leq 360$.	2

Give 1 mark for each •

Illustrations for awarding each •

<p>a ans: $2\sin(x + 30)^\circ$ 4 marks</p> <ul style="list-style-type: none"> •¹ ss : expand $k\sin(x + a)^\circ$ •² ic : compare coefficients •³ pd : process k •⁴ pd : process a 	<ul style="list-style-type: none"> •¹ $k \sin x^\circ \cos a^\circ + k \cos x^\circ \sin a^\circ$ stated explicitly •² $k \sin a^\circ = 1$ and $k \cos a^\circ = \sqrt{3}$ stated explicitly •³ $k = 2$ •⁴ $a = 30$
<p>b ans: max = 7 when $x = 60$ 2 marks</p> <ul style="list-style-type: none"> •⁵ ic : interpret maximum •⁶ sp : know how to and find x_{\max} 	<ul style="list-style-type: none"> •⁵ max = $5 + 2 = 7$ •⁶ $x = 60$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
4		2.3.5	CN	C	2002W q5

Solve the equation $\cos 2x - 2\sin^2 x = 0$ in the interval $0 \leq x < 2\pi$.

4

Give 1 mark for each •

Illustrations for awarding each •

ans: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

4 marks

- ¹ ss : know to make angles same size
- ² pd : simplify
- ³ pd : process solution
- ⁴ pd : process solution

Method 1

•¹ use eg $2\sin^2 x = 1 - \cos 2x$

•² $\cos 2x = \frac{1}{2}$

•³ $2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

•⁴ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Alternative for •³ and •⁴

•³ $2x = \frac{\pi}{3}, \frac{5\pi}{3}$ **and** $x = \frac{\pi}{6}, \frac{5\pi}{6}$

•⁴ $2x = \frac{7\pi}{3}, \frac{11\pi}{3}$ **and** $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Method 2

•¹ use eg $\cos 2x = 1 - 2\sin^2 x$

•² $\sin^2 x = \frac{1}{4}$

•³ $\sin x = \frac{1}{2} \dots \dots x = \frac{\pi}{6}, \frac{5\pi}{6}$

•⁴ $\sin x = -\frac{1}{2} \dots \dots x = \frac{7\pi}{6}, \frac{11\pi}{6}$

•³ $\sin x = \frac{1}{2} \dots \dots x = \frac{\pi}{6}$

and $\sin x = -\frac{1}{2} \dots \dots x = \frac{7\pi}{6}$

•⁴ $\frac{5\pi}{6}, \frac{11\pi}{6}$

Method 3

•¹ use eg $\cos 2x = \cos^2 x - \sin^2 x$

•² $\tan^2 x = \frac{1}{3}$

•³ $\tan x = \frac{1}{\sqrt{3}} \dots \dots x = \frac{\pi}{6}, \frac{7\pi}{6}$

•⁴ $\tan x = -\frac{1}{\sqrt{3}} \dots \dots x = \frac{5\pi}{6}, \frac{11\pi}{6}$

Alternative for •³ and •⁴

similar to method 2

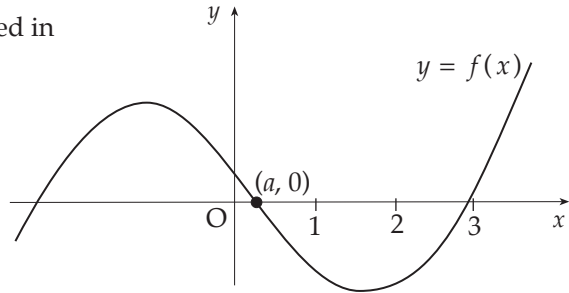
Notes

- 1 answers left in degrees lose 1 mark
ie if a candidate gets to 30° and 150° and does no more then it only earns •¹ and •².

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	2.1.11	CR	C	2002W q6

The graph of $f(x) = 2x^3 - 5x^2 - 3x + 1$ has been sketched in the diagram shown.

Find the value of a correct to one decimal place.



3

Give 1 mark for each •

Illustrations for awarding each •

ans: $a = 0.2$

3 marks

- ¹ ss : find a \pm ve $f(b)$ for suitable b
- ² ss : find a \pm ve $f(c)$ for suitable c and another value of $f(x)$ for $c < x < b$.
- ³ pd : process minimum requirements

- ¹ evaluating $f(b)$ for any b making $f(b) + ve$ or $-ve$, $0 \leq b \leq 0.5$
- ² evaluating $f(c)$ for any c making $f(c) - ve$ or $+ve$, $0 \leq c \leq 0.5$ **and** evaluating $f(d)$ for any d lying between b and c .
- ³ $a = 0.2$

Notes

- 1 The minimum requirements to conclude that $a = 0.2$ is to show that $0.2 < a < 0.25$

x	$f(x)$
0	1
0.1	0.625
0.2	0.216
0.25	-0.03125
0.3	-0.296
0.4	-0.872
0.5	-1.5
1	-5

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	3.2.4	CR	B	2002W q7

Find $\int_0^1 \left(\cos(3x) - \sin\left(\frac{1}{3}x + 1\right) \right) dx$ correct to 3 decimal places.

3

Give 1 mark for each •

Illustrations for awarding each •

ans: $a = -0.868$

3 marks

- ¹ ic : integrate $\cos ax$
- ² ic : integrate $\sin(ax+b)$
- ³ pd : evaluate limits

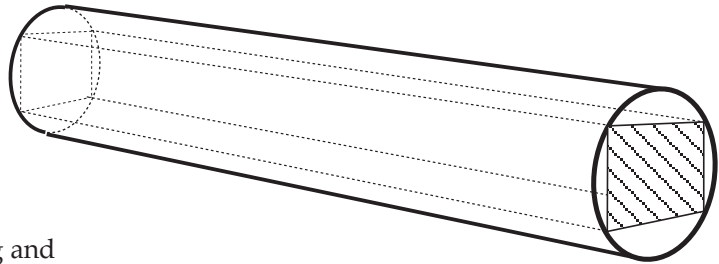
- ¹ $\frac{1}{3} \sin 3x$
- ² $+\frac{1}{\frac{1}{3}} \cos\left(\frac{1}{3}x + 1\right)$
- ³ $\left(\frac{1}{3} \sin 3 + 3 \cos \frac{4}{3}\right) - 3 \cos 1 = -0.868$

Note

$$1 \quad 0.047 + 0.706 - 1.62 = -0.868$$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	5	1.3.15	CR	B	2002W q8

A rectangular beam is to be cut from a cylindrical log of diameter 20 cm.

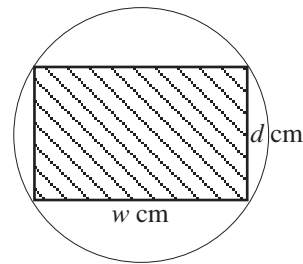


The diagram shows a cross-section of the log and beam where the beam has a breadth of w cm and a depth of d cm.

The strength S of the beam is given by

$$S = 1.7w(400 - w^2).$$

Find the dimensions of the beam for maximum strength.



5

Give 1 mark for each •

Illustrations for awarding each •

ans: $d = 20\sqrt{\frac{2}{3}}$

5 marks

- ¹ ss : know to set derivative to zero
- ² pd : differentiate
- ³ pd : solve
- ⁴ ic : justify maximum
- ⁵ pd : substitute

- ¹ $\frac{dS}{dw} = \dots = 0$ **stated explicitly**
- ² $680 - 5.1w^2$
- ³ $w = \frac{20}{\sqrt{3}}$ (11.5)
- ⁴ e.g. nature table
- ⁵ $d = 20\sqrt{\frac{2}{3}}$ (16.3)

Note

1 " $680 - 5.1w^2 = 0$ " would earn •¹ and •².

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	1	3.3.4	CR	C	2002W q9
b)	4	3.3.4	CR	B	

A researcher modelled the size N of a colony of bacteria t hours after the beginning of her observations by $N(t) = 950 \times (2 \cdot 6)^{0.2t}$.

- (a) What was the size of the colony when observations began? 1
- (b) How long does it take for the size of the colony to be multiplied by 10? 4

Give 1 mark for each •

Illustrations for awarding each •

a ans: 950 1 mark

•¹ ic : interpret $N(0)$

•¹ 950

b ans: approx 12 hours 4 marks

•² ic : substitute

•² $9500 = 950 \times 2.6^{0.2t}$

OR $10 = 2.6^{0.2t}$

•³ ss : know how to isolate index

•³ $\log_{10} 10 = \log_{10} 2.6^{0.2t}$

•⁴ pd : process solution

•⁴ $0.2t = \frac{\log_{10} 10}{\log_{10} 2.6}$

•⁵ pd : process solution

•⁵ $t \approx 12$ hours

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	7	2.4.4, 2.1.8	CN	A	2002W q10
b)	2	2.4.4	CN	A	

The line $y + 2x = k$, $k > 0$, is a tangent to the circle $x^2 + y^2 - 2x - 4 = 0$.

- | | |
|---|---|
| (a) Find the value of k . | 7 |
| (b) Deduce the coordinates of the point of contact. | 2 |

Give 1 mark for each •

Illustrations for awarding each •

a ans: $k = 7$ 7 marks

- ¹ ss : know to sub and start process
- ² ic : substitute
- ³ pd : simplify to quadratic
- ⁴ ss : know to make discriminant zero
- ⁵ ic : identify discriminant
- ⁶ pd : simplify to quadratic
- ⁷ pd : solve

- ¹ $y = k - 2x$ stated or implied by •²
- ² $x^2 + (k - 2x)^2 - 2x - 4 = 0$
- ³ $5x^2 - (2 + 4k)x + (k^2 - 4) = 0$
- ⁴ discriminant = 0 stated or implied by •⁵
- ⁵ $(2 + 4k)^2 - 4 \times 5 \times (k^2 - 4) = 0$
- ⁶ $-4k^2 + 16k + 84 = 0$
- ⁷ $k = 7$

b ans: (3, 1) 2 marks

- ⁸ ss : know to substitute
 - ⁹ pd : solve
- ⁸ $5x^2 - 30x + 45 = 0$
 - ⁹ (3,1)

Notes

- 1 The zero at •²/•³ stage must appear at least once.
- 2 The zero at •⁴/•⁵/•⁶ stage must appear at least once.
- 3 Substitution at •¹ can be for x
- 4 If a value for k is not found in (a), a plucked k may be used in (b) with no loss of available marks.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	7	2.2.7	CN	A	2002W q11
b)	1	2.2.7	CN	A	

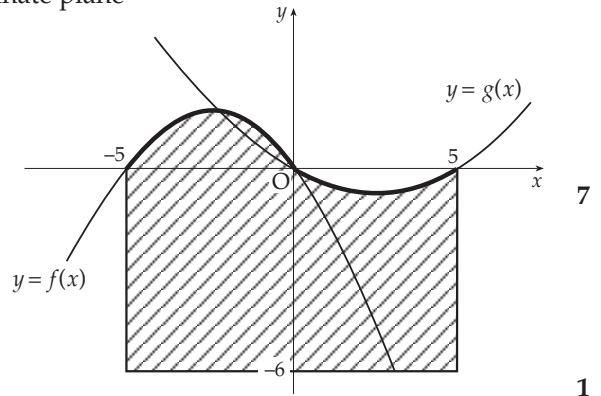
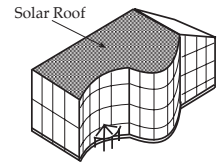
An energy efficient building is designed with solar cells covering the whole of its south facing roof. The energy generated by the solar cells is directly proportional to the area in square units.

The shape of the solar roof can be represented on the coordinate plane as the shaded area bounded by the functions

$f(x) = \frac{1}{4}(-x^2 - 5x)$, $g(x) = \frac{1}{12}(x^2 - 5x)$ and the lines $x = -5$, $x = 5$ and $y = -6$.

- (a) Find the area of the solar roof.
- (b) Ten square units of solar cells generate a maximum of 1 kilowatt.

What is the maximum energy the solar roof can generate in kilowatts (to the nearest kilowatt)?



Give 1 mark for each •

Illustrations for awarding each •

a ans: $63 \frac{17}{36}$ 7 marks

- ¹ ss : split into calculatable parts
- ² ic : first integral with limits
- ³ pd : integrate
- ⁴ pd : process limits
- ⁵ ic : second integral with limits
- ⁶ pd : integrate
- ⁷ ic : process limits & complete strategy

Method 1

- ¹ strategy : f to x -axis + g to x -axis + rectangle
- ² $\int_{-5}^0 \frac{1}{4}(-x^2 - 5x) dx$
- ³ $\left[-\frac{1}{12}x^3 - \frac{5}{8}x^2\right]_{-5}^0$
- ⁴ $5 \frac{5}{24}$ (5.2)
- ⁵ $\int_0^5 \frac{1}{12}(x^2 - 5x) dx$
- ⁶ $\left[\frac{1}{36}x^3 - \frac{5}{24}x^2\right]_0^5$
- ⁷ $-1 \frac{53}{72}$ (-1.7) and $63 \frac{17}{36}$ (63.5)

Note •², •³, •⁴ apply to the first integral, whichever it is.

b ans: 6 kw 1 mark

- ⁸ ic : interpret context

•⁸ 6 kilowatts

Method 2

-
- ¹ strategy : $(y = f) - (y = -6) + (y = g) - (y = -6)$
 - ² $\int_{-5}^0 \frac{1}{4}(-x^2 - 5x) - (-6) dx$
 - ³ $\left[-\frac{1}{12}x^3 - \frac{5}{8}x^2 + 6x\right]_{-5}^0$
 - ⁴ $35 \frac{5}{24}$ (35.2)
 - ⁵ $\int_0^5 \frac{1}{12}(x^2 - 5x) - (-6) dx$
 - ⁶ $\left[\frac{1}{36}x^3 - \frac{5}{24}x^2 + 6x\right]_0^5$
 - ⁷ $28 \frac{19}{72}$ (28.3) and $63 \frac{17}{36}$ (63.5)