W100/301

NATIONAL QUALIFICATIONS 2002

FRIDAY, 18 JANUARY 9.00 AM – 10.10 AM

MATHEMATICS HIGHER

Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**
2 Full credit will be given only where the solution contains appropriate working.
3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

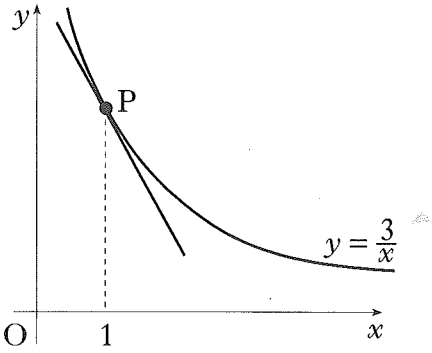
1. (*a*) Find the equation of the straight line through the points A(−1, 5) and B(3, 1). 2

   (*b*) Find the size of the angle which AB makes with the positive direction of the x-axis. 2

2. (*a*) If $\boldsymbol{u} = \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, write down the components of $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$. 2

   (*b*) Hence, or otherwise, show that $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$ are perpendicular. 2

3. Find the equation of the tangent to the curve with equation $y = \frac{3}{x}$ at the point P where $x = 1$. 5



4. (*a*) Write down the exact values of $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$. 1

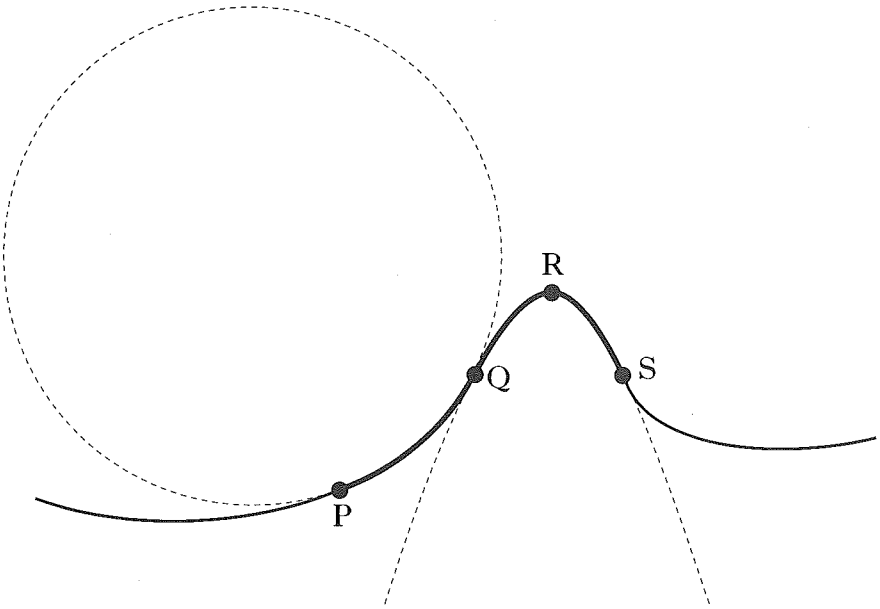
   (*b*) If $\tan x = 4\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$, find the exact values of x for $0 \leq x \leq 2\pi$. 2

5. Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d. 5

[Turn over

Marks

6. The side view of part of a roller coaster ride is shown by the path PQRS.
The curve PQ is an arc of the circle with equation $x^2 + y^2 + 4x - 10y + 9 = 0$.
The curve QRS is part of the parabola with equation $y = -x^2 + 6x - 5$.
The point Q has coordinates (2, 3).



(*a*) Find the equation of the tangent to the circle at Q. **4**

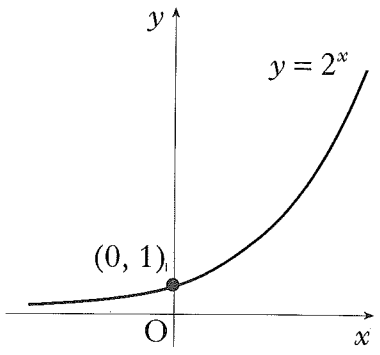
(*b*) Show that this tangent to the circle at Q is also the tangent to the parabola at Q. **2**

7. Find $\int\left(\sqrt[3]{x} - \frac{1}{\sqrt{x}}\right)dx$. **4**

8. The diagram shows part of the graph of $y = 2^x$.

(*a*) Sketch the graph of $y = 2^{-x} - 8$. **2**

(*b*) Find the coordinates of the points where it crosses the x and y axes. **2**



Marks

9. The function f, defined on a suitable domain, is given by $f(x) = \frac{3}{x+1}$.

(*a*) Find an expression for $h(x)$ where $h(x) = f(f(x))$, giving your answer as a fraction in its simplest form. **3**

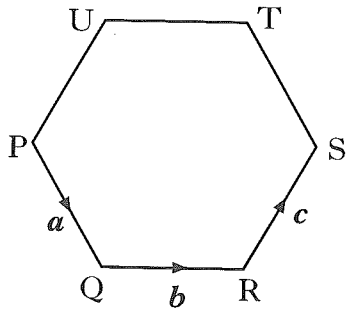
(*b*) Describe any restriction on the domain of h. **1**

10. A function f is defined by $f(x) = 2x + 3 + \frac{18}{x-4}$, $x \neq 4$.

Find the values of x for which the function is increasing. **5**

11. PQRSTU is a regular hexagon of side 2 units. $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RS}$ represent vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively.

Find the value of $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c})$. **3**



12. If $\log_a p = \cos^2 x$ and $\log_a r = \sin^2 x$, show that $pr = a$. **3**

[END OF QUESTION PAPER]